Comparisons of anomalous and collisional radial transport with a continuum kinetic edge code

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as part of

edge
simulation
laboratory

Abstract

Modeling of anomalous (turbulence-driven) radial transport in controlled-fusion plasmas is necessary for long-time transport simulations. Here the focus is continuum kinetic edge codes such as the (2-D, 2-V) transport version of TEMPEST, NEO, and the code being developed by the Edge Simulation Laboratory, but the model also has wider application. Our previously developed anomalous diagonal transport matrix model with velocity-dependent convection and diffusion coefficients allows contact with typical fluid transport models (e.g., UEDGE). Results are presented that combine the anomalous transport model and collisional transport owing to ion drift orbits utilizing a Krook collision operator that conserves density and energy. Comparison is made of the relative magnitudes and possible synergistic effects of the two processes for typical tokamak device parameters.

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Overview

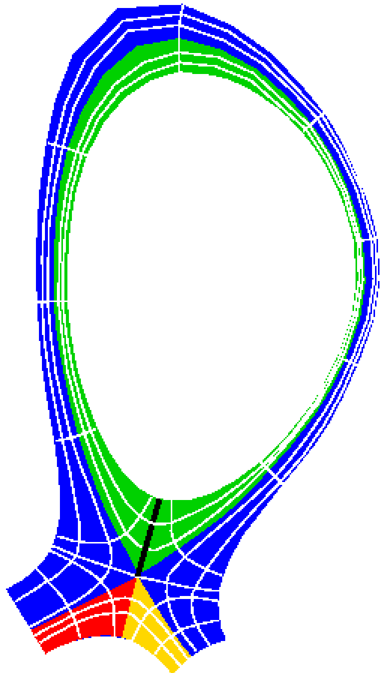
- TEMPEST is a 5-D kinetic code to simulate edge plasmas; runnable as a 4-D kinetic transport code.

- In order to perform quick-running studies of combined neoclassical and anomalous transport, we have added the following to TEMPEST:
  - a model for anomalous transport
  - an upgraded Krook collision model that simultaneously conserves density and energy

- This approach provides a possible starting point for a future self- consistent turbulence and transport model, where the model transport coefficients would be extracted from a simultaneously running 5D simulation.

Gyrokinetic equation has been implemented in the continuum TEMPEST for the edge

$$
\begin{aligned}
\frac{\partial F_\alpha}{\partial t} &+ \bar{\mathrm{v}}_{\mathrm{d}} \cdot \nabla_\perp F_\alpha + (\bar{v}_{\parallel\alpha} + v_{Banos})\nabla_\parallel \partial F_\alpha \\
&+ \left[q\frac{\partial \langle \Phi_0 \rangle}{\partial t} + \bar{\mu}\frac{\partial B}{\partial t} - \frac{qB}{B^*}\bar{v}_\parallel \nabla_\parallel \langle \delta\phi \rangle - q\mathrm{v}_{\mathrm{d}}^0 \cdot \bar{\nabla}\langle \delta\phi \rangle \right] \frac{\partial F_\alpha}{\partial E_0} \\
&= C(F_\alpha, F_\alpha),
\end{aligned}
$$



- GK F-equation discretized with high order (4th); Fokker-Planck collisions
- Full-f and δf options available
- Circular & divertor geom.; 2D equilibrium potential
- Runnable as
  - 4-D for transport with F(Ψ,θ,ε,μ), or
  - 5-D for turbulence with F(Ψ,θ,ϕ ,ε,μ)
- Extensions planned:
  - sources/sinks
  - model transport coefficients for initial anomalous transp.
  - generalized GK equations (see Qin)
  - optional fluid equations in same framework
  - *field-aligned coordinates for evolving B

This poster

Formulation, implementation, and testing of an anomalous radial diffusion operator

Our goal is to add a model for turbulent transport that can be combined with neoclassical ion transport

- **presently a diagonal transport matrix for comparison with fluid models**
- **longer-term goal: match arbitrary transport matrix**

Model the turbulent transport as a combination of advection and diffusion, as is conventionally done in fluids

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial s} + v_d \cdot \nabla f + E_\parallel \frac{\partial f}{\partial v_\parallel} + \frac{1}{V}\frac{\partial\left(V\,\Gamma_a/h_\psi\right)}{\partial \psi}\bigg|_{\theta,v} = S$$

$$\Gamma_a = U_a f - \frac{D}{h_\psi}\frac{\partial f}{\partial \psi}\bigg|_{\theta,v}$$

– U_a, D depend on position (ψ) and velocity (v)

- specifying different velocity dependence allows separate control of "D_n" and "χ" (and mom. coeff.)
- advection U_a allows $D(v)$ to be positive for all velocities
- provides flexibility, speed (compared to coupling turbulence), and comparison to fluid models

Modeling the transport coefficients

Define convective coefficient $$U_a(\psi) = \frac{1}{h_\psi}\left(\alpha\frac{\partial \ln n}{\partial \psi} + \beta\frac{\partial \ln T}{\partial \psi}\right)$$

Define diffusive coefficient $$D(\psi, \hat{\mathrm{v}}) = D_0 + D_2\left(\hat{\mathrm{v}}^2 - \tfrac{1}{2}\right) + D_4\left(\hat{\mathrm{v}}^4 - 3\hat{\mathrm{v}}^2 + \tfrac{3}{4}\right)$$

where $\left(\hat{\mathrm{v}} = \mathrm{v}/\mathrm{v}_{th}\right)$

Particle flux $$h_\psi \Gamma_n = -(D_0 + D_2 - \alpha)\frac{\partial n}{\partial \psi} - \frac{3}{2}\left(D_2 + 2D_4 - \frac{2}{3}\beta\right) n \frac{\partial \ln T}{\partial \psi}$$

&

Heat flux $$h_\psi Q = -\frac{3}{2}\left\{(D_0 + 2D_2 + 2D_4 - \alpha) T \frac{\partial n}{\partial \psi} + \left(D_0 + \frac{9}{2}D_2 + 12D_4 - \beta\right) n \frac{\partial T}{\partial \psi}\right\}$$

Modeling the transport coefficients

$$\boxed{\begin{aligned} h_\psi \Gamma_n &= -(D_0 + D_2 - \alpha)\frac{\partial n}{\partial \psi} - \frac{3}{2}\left(D_2 + 2D_4 - \frac{2}{3}\beta\right) n \frac{\partial \ln T}{\partial \psi} \\ h_\psi Q &= -\frac{3}{2}\left\{(D_0 + 2D_2 + 2D_4 - \alpha) T \frac{\partial n}{\partial \psi} + \left(D_0 + \frac{9}{2}D_2 + 12D_4 - \beta\right) n \frac{\partial T}{\partial \psi}\right\} \end{aligned}}$$

Diagonal form of the transport matrix

- Particle flux not directly dependent on the local temperature gradients
- Particle flux leads to a corresponding heat flux (specific heat)

$$\boxed{\begin{aligned} &\beta = D_n && D_4 = \frac{1}{5}\left(\frac{2}{3}\chi - \frac{4}{3}\beta - D_n - \alpha\right) \\ &D_2 = 2\left(\frac{1}{3}\beta - D_4\right) && D_0 = D_n + \alpha - D_2 \end{aligned}}$$

- For the diffusion coefficient with a simple quadratic dependence over speed (v), we choose $D_4 = 0$
- Diffusion coefficient is non-negative over the velocity domain if $\alpha \geq 0$

$$\boxed{\begin{aligned} D(\psi, \hat{\mathrm{v}}) &= \frac{2}{3} D_n \hat{\mathrm{v}}^2 + \alpha \\ h_\psi U_a &= \alpha \frac{\partial \ln n}{\partial \psi} + D_n \frac{\partial \ln T}{\partial \psi} \end{aligned}} \longrightarrow \boxed{\begin{aligned} h_\psi \Gamma_n &= -D_n \frac{\partial n}{\partial \psi} \\ h_\psi Q &= -\frac{5}{2} D_n T \frac{\partial n}{\partial \psi} - \chi n \frac{\partial T}{\partial \psi} \end{aligned}}$$

Anomalous radial transport: Implementation

- Velocity coordinates are (ε_o, μ)
  - derivative at constant **v** not the same as derivative at constant (ε_o, μ)

$$\left.\frac{\partial f}{\partial \psi}\right|_{\theta,\mathrm{v}} = \frac{f(\psi+\delta\psi,\mu_+) - f(\psi-\delta\psi,\mu_-)}{2\,\delta\psi}$$

$$\mu_+ = \frac{\mu B(\psi)}{B(\psi+\delta\psi)} \qquad \mu_- = \frac{\mu B(\psi)}{B(\psi-\delta\psi)}$$

  - The interpolation along μ is performed using polynomial interpolation over the adjacent μ cells

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

- Kinetic equation is now second-order differential equation in space
  - boundary conditions enforced for incoming & outgoing particles at radial boundaries
  - diffusion evaluated using a 2nd order Central Differencing Scheme
- Contribution to radial transport computed by computing moments of the flux (Γ_a) over the velocity (ε_o, μ) space

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Krook Collision Model: Implementation

Computation of radial transport of particles and energy requires a number- and energy-conserving collision model. In order to have one that is reasonably fast, we implement an upgrade of TEMPEST's Krook model that simultaneously conserves energy and particle density

$$\left.\frac{\partial f}{\partial t}\right|_c = -\nu_k\left[f(\mathrm{v}) - f_{Max}(\mathrm{v})\right]$$

- f_{Max} is a Maxwellian corresponding to the local density and temperature
- model conserves energy & particle density

$$\int d\mathbf{v}\, f(\mathbf{v}) = \int d\mathbf{v}\, f_M(\mathbf{v})$$
$$\int d\mathbf{v}\, \mathrm{v}^2 f(\mathbf{v}) = \int d\mathbf{v}\, \mathrm{v}^2 f_M(\mathbf{v})$$
$$\left.\frac{\partial n}{\partial t}\right|_c = -\nu_c \int d\mathbf{v}\left[f(\mathbf{v}) - f_{Max}(\mathbf{v})\right] = 0$$
$$\left.\frac{\partial}{\partial t}(nT)\right|_c = -\nu_c \int d\mathbf{v}\, \mathrm{v}^2\left[f(\mathbf{v}) - f_{Max}(\mathbf{v})\right] = 0$$

Moments of the distribution function are needed to update the Maxwellian

- Numerical error in the moment computation affects f_{Max} and hence, the conservative character of the collision term

$$(n,T) \rightarrow f(\mathrm{v}) \xrightarrow{\text{num}} (n_1,T_1) \rightarrow f_M(\mathrm{v}) \xrightarrow{\text{num}} (n_2,T_2)$$
$$(n,T) \neq (n_1,T_1) \neq (n_2,T_2)$$

- We avoid this by defining the local Maxwellian as a linear combination of two Maxwellians of known numerical moments

Computed only at t=0

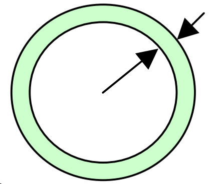
at current time-step

$$f_M(\mathrm{v}) = a_1 f_{M_1}(\mathrm{v}) + a_2 f_{M_2}(\mathrm{v})$$
$$f_{M_i}(\mathrm{v}) \xrightarrow{\text{num}} (n_i,T_i)$$
$$f(\mathrm{v}) \xrightarrow{\text{num}} (n,T)$$

$$a_1 = \frac{n}{n_1}\left(\frac{T_2 - T}{T_2 - T_1}\right); \qquad a_2 = \frac{n}{n_2}\left(\frac{T - T_1}{T_2 - T_1}\right)$$
$$f_M(\mathrm{v}) \xrightarrow{\text{num}} (n,T) \xleftarrow{num} f(\mathrm{v})$$

Anomalous Diffusion + Krook Collision: Test case

- Diffusion and collision model implementations were tested on an annular geometry
  - Non-uniform magnetic field → annulus in a tokamak geometry
  - domain is periodic in the poloidal direction
  - Krook collision term is computed to check conservation/cost
  - initially radial/poloidal drift switched off → diffusion in an annulus



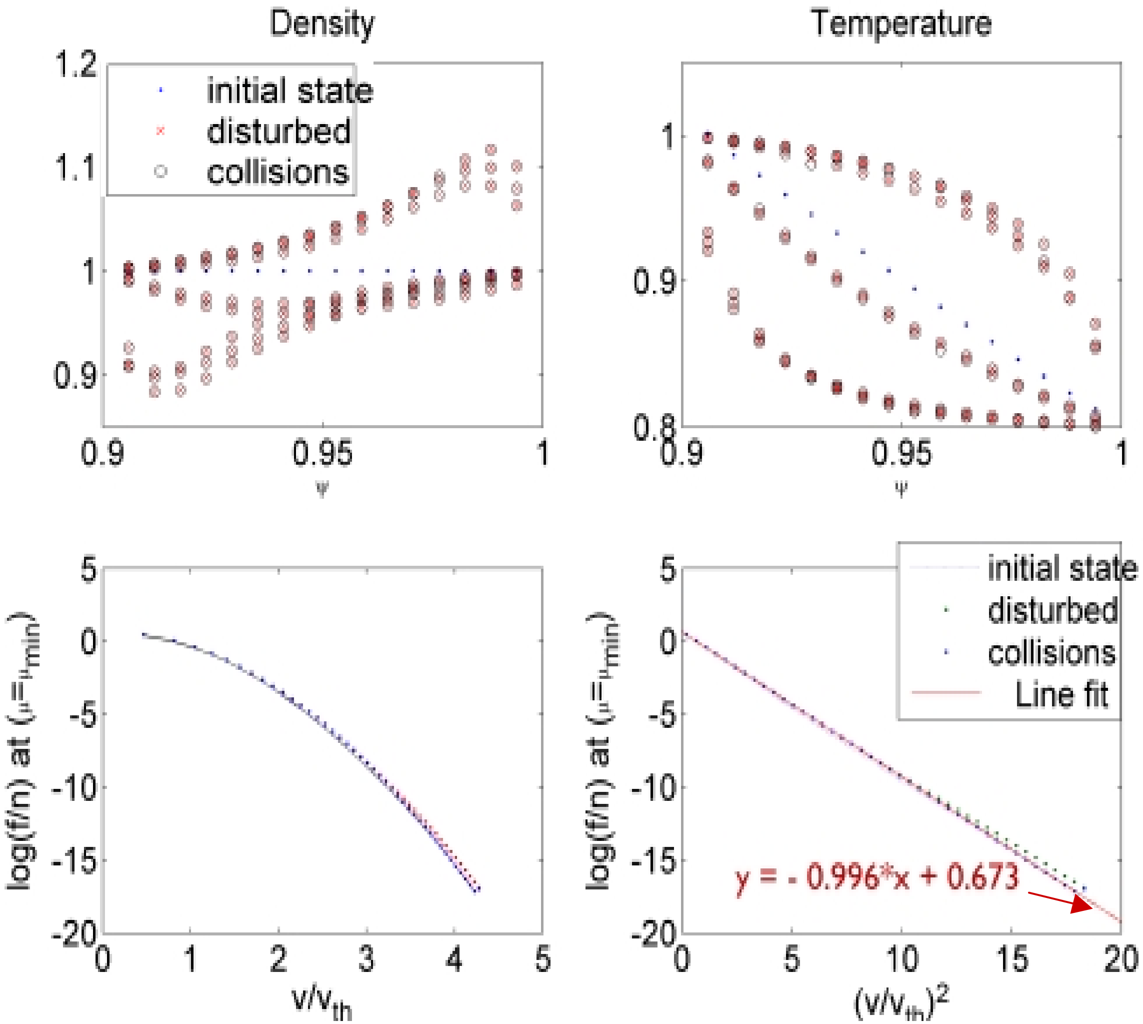
$$\frac{\partial f}{\partial t} = -\frac{1}{V}\frac{\partial}{\partial \psi}\left[\frac{V}{h_\psi}\frac{D(\psi,\mathrm{v})}{h_\psi}\frac{\partial f}{\partial \psi}\right] - \upsilon_k\left[f(\mathrm{v}) - f_{Max}(\mathrm{v})\right]$$

$$V = 2\pi R h_\psi h_\theta$$

  - **Case I**: Anomalous transport model, without Krook collision model
  - **Case II**: Krook collision model
- Simulation parameters:
  - domain width is 0.1*minor radius
  - spatial grid: 16(radial) by 8(poloidal)
  - Max kinetic energy (velocity space extent): 16*T
  - Velocity space grid: (ε_o, μ) mesh size = 42x65

I. Krook collision model

- Initial state is of uniform density with radial temperature gradient
- Particle drifts disturb the distribution function from it's initial state
- Krook collision model was then run on the disturbed state
- Krook conserves particles and energy (plots at the top)
- Krook relaxes the distribution function to a Maxwellian state



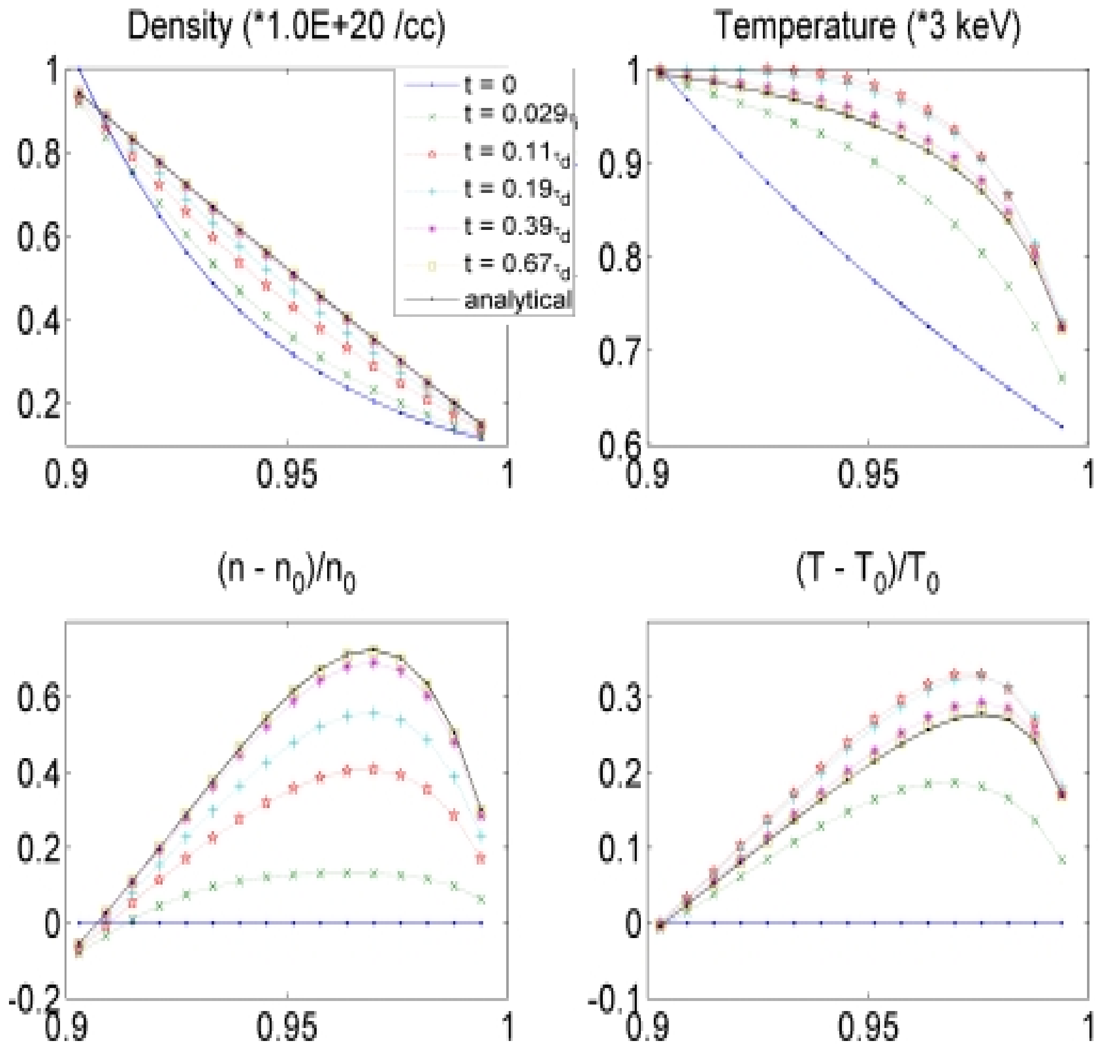
II. Anomalous transport model

- Initial state with radial density and temperature gradients
- Magnetic field is uniform
- Diffusion model is defined by a diffusivity (D_n) of 10 m^2/s and conductivity(χ) of 35 m^2/s

$$D(\psi, \mathrm{v}) = D_n \, \mathrm{v}^2 / \mathrm{v}_{\mathrm{th}}^2$$

$$h_\psi U_a = D_n \frac{\partial \ln T}{\partial \psi}$$

- Density, Temperature are initialized as exponential variations
  - n_0, T_0 are the density and temperature at t=0
- Analytical solution computed using equations for density/temperature evolution outside TEMPEST



Normalization time scale is $\tau_d = \Delta^2 / D_n$

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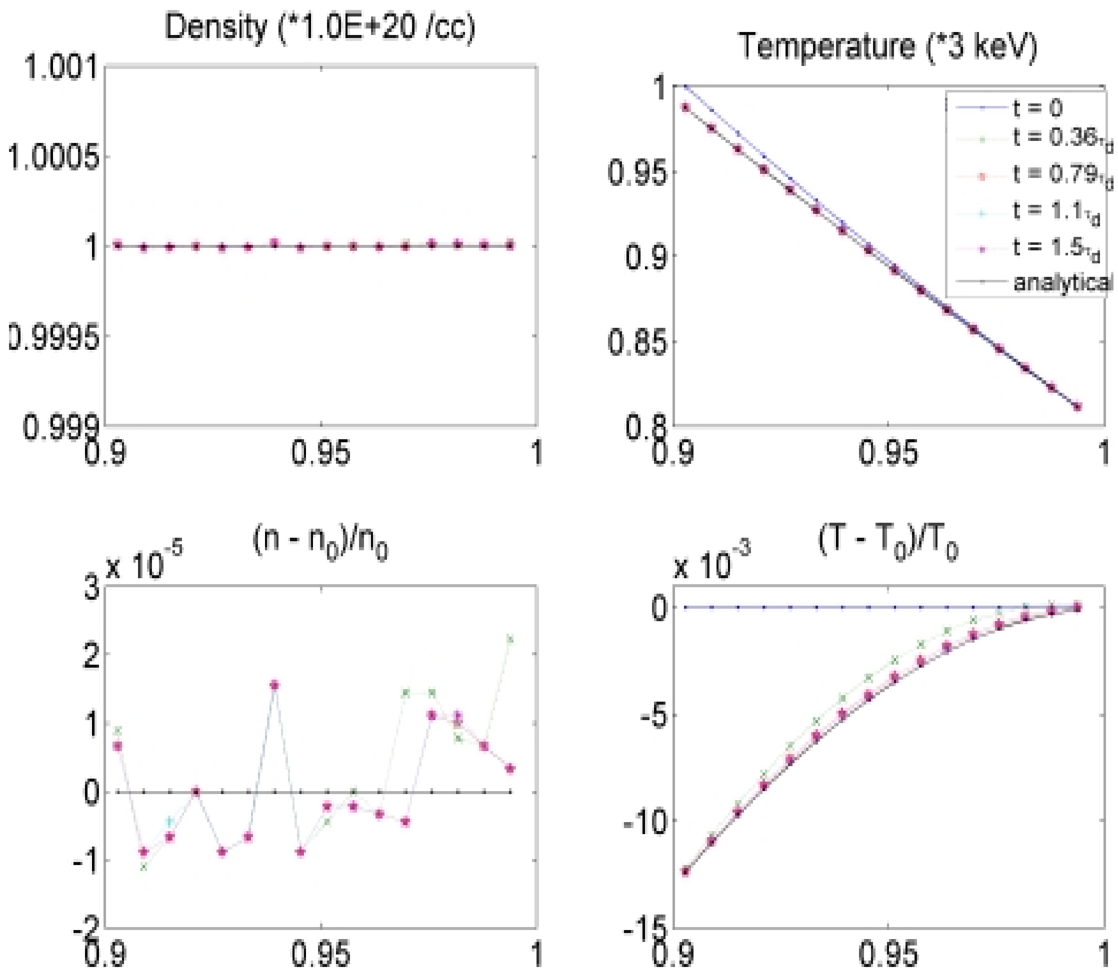
III. Anomalous transport model

- Initial state with uniform density and a radial temperature gradient
- Non-uniform magnetic field
  - annulus in a large aspect-ratio tokamak
- Diffusion model is defined by a diffusivity (D_n) of 10 m²/s

$$D(\psi, \mathrm{v}) = D_n$$

$$h_\psi U_a = 0$$

- Conductivity(χ) is of the same magnitude as diffusivity for this model



IV. Anomalous transport model

– Initial state with radial density and temperature gradients

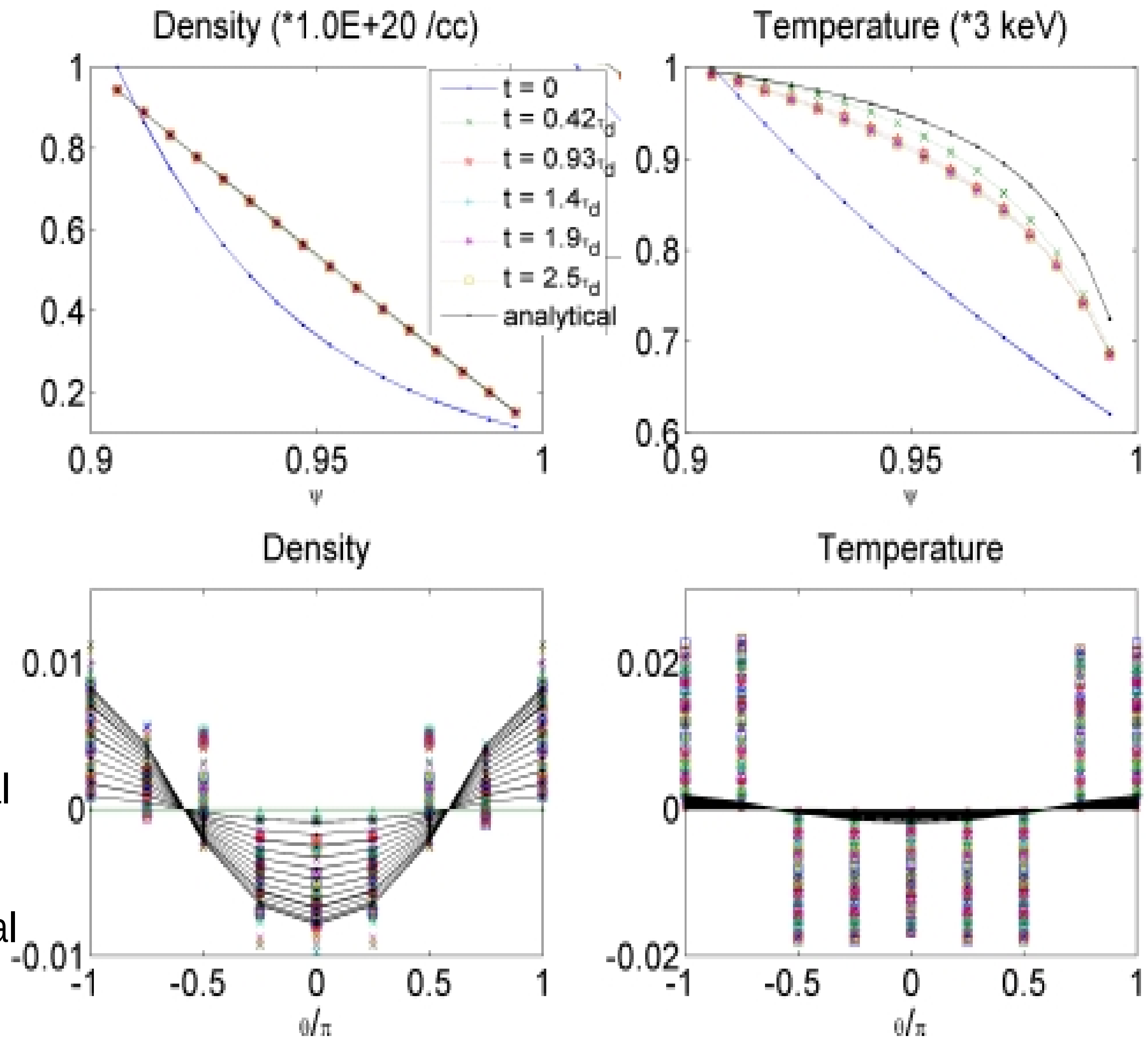
– Non-uniform magnetic field
  - annulus in a tokamak

– Diffusion model is defined by a diffusivity (D_n) of 10 m^2/s and conductivity(χ) of 35 m^2/s

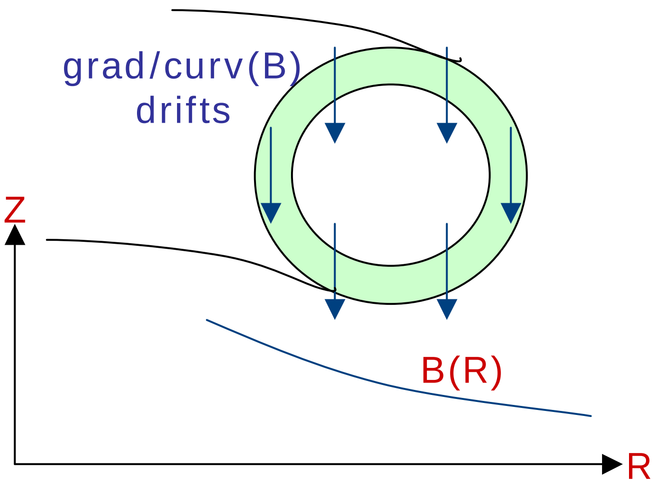
$$D(\psi, \mathrm{v}) = D_n \, \mathrm{v}^2 / \mathrm{v}_{\mathrm{th}}^2$$

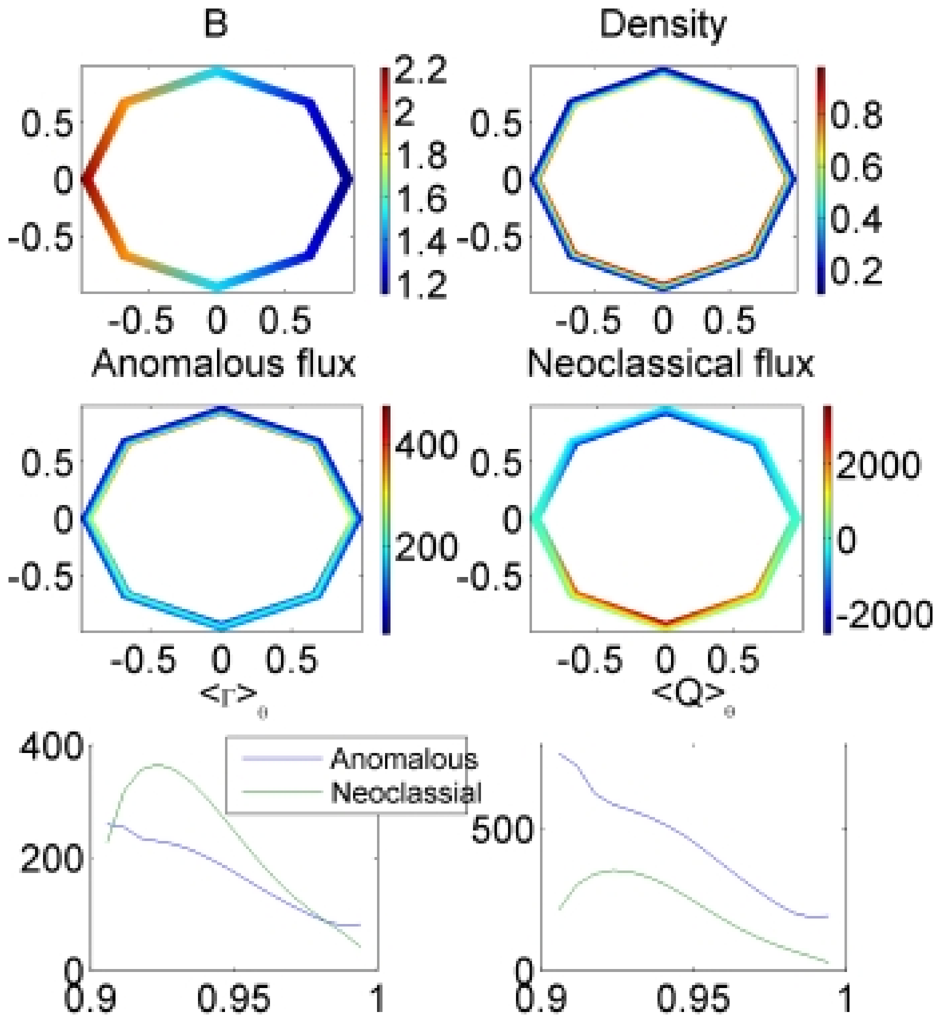
$$h_\psi U_a = D_n \frac{\partial \ln T}{\partial \psi}$$

– Plots at the top row show radial variation of poloidally averaged values; bottom row plots show poloidal variation about the local poloidal mean
  - Density variation agrees with analytical expectation
  - Temperature variation does not match analytical expectation exactly



Neoclassical & Anomalous radial transport





- In this test simulation, we combine the anomalous transport model with radial drifts and the Krook collision model. There is no radial electric field in this case, and hence the drifts are due to grad(B) and curv(B).
- We can see that, though the flux due to the drifts is much larger locally, the poloidally averaged radial transport due to the drifts is of the same order as anomalous transport.

Summary

- An anomalous radial transport model in the form of a combination of convective and diffusive transport has been added to TEMPEST
  - the model is in the form of a diagonal transport matrix; transport coefficients can be assigned so as to be equivalent in the highly collisional limit to fluid models; allows comparison with fluid models
  - simulations done using the model compare well with the analytical expectations for varying density and temperature in a ring geometry
  - in the future the transport coefficients could be extracted from a simultaneously running 5D simulation
- The Krook collision model in TEMPEST has been upgraded to make it more suitable for simultaneous neoclassical and anomalous radial transport calculations
  - Krook model conserves particles and energy