Unified Gyrokinetic Simulations of Drift Wave Turbulence and Neoclassical Transport

E. A. Belli*, J. Candy, P. B. Snyder

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Abstract

A 5D δf Eulerian gyrokinetic code (EGK) has been developed as a rapid prototype code for the Edge Simulation Laboratory. The purpose of this code is to explore numerical issues associated with the (μ, $v_{||}$) velocity space formulation for gyrokinetics and to study physical effects associated with extensions to full F. EGK has been successfully benchmarked for ITG/TEM linear drift wave physics and the collisionless damping of the zonal flow potential, including kinetic electrons. Recently, a version of EGK, which solves the vorticity Poisson constraint equation, has also performed successful simulations of neoclassical ion transport, including the self-consistent radial electric field, neglecting the poloidal variation of Φ. Using pitch angle scattering collisions and assuming the flux tube limit, this simplified code has reproduced the saturated E_r results of Satake et al. (Nuclear Fusion **45**, 1362 (2005)), who used a radially global simulation. Here we present new results which extend these studies of neoclassical transport to include the effects of the poloidal variation of E_r and kinetic electron dynamics. Development of a unified, global EGK code which solves drift wave physics and neoclassical physics using the same algorithms is also discussed.

EGK is a prototype gyrokinetic code for ESL based on the (μ ,$v_{||}$) velocity space formulation.

Motivation:

- EGK is being developed first as a δf code to explore numerical issues and dissipation algorithms for the (μ ,$v_{||}$) gyrokinetic formulation.
- Subsequent studies will focus on physical effects associated with extensions to full F.

Physics Studies/Development:

1) δf linear, electrostatic gyrokinetic simulations including gk electrons

Status: Successfully benchmarked for ITG/TEM linear drift wave physics and collisionless damping of zonal flows

2) Neoclassical simulations with self-consistent E_r

Using the radially local limit and pitch angle scattering collisions

a) Neglecting the poloidal variation of Φ

Status: Successfully benchmarked against radially global simulations and standard neoclassical theory

b) Including $\Phi(\theta)$ and kinetic electron dynamics

Status: Initial studies complete; qualitative comparisons with theory.

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3) Unified global simulations of drift wave turbulence + neoclassical transport.

I. Linear Gyrokinetic Studies

EGK presently solves the linear $\delta f(R,\mu,v_{||})$ gyrokinetic equations in the electrostatic limit:

$$\frac{\partial f}{\partial t}+\left(v_{\|}\hat{b}\cdot\nabla+\vec{v}_d\cdot\nabla\boxed{-\mu\hat{b}\cdot\nabla B\frac{\partial}{\partial v_{\|}}}\right)f-C(f)=\frac{Ze}{T_0}F_0\left(\vec{v}_*\cdot\nabla-v_{\|}\hat{b}\cdot\nabla-\vec{v}_d\cdot\nabla\right)J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)\Phi$$

$$\sum_s\frac{Z_s^2e^2n_{0s}}{T_{0s}}(1-\Gamma_{0s})\Phi=\sum_s Z_s e\int d^3v J_{0s}\left(\frac{k_\perp v_\perp}{\Omega}\right)f_s$$

Why explore $(\mu,v_{||})$ velocity space coordinates?

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In comparison with (E,λ) (used in GS2[1] and GYRO[2])
or (E,μ) (used in the full F code TEMPEST),
$(\mu,v_{||})$ coordinates allow for:

- **a simpler volume element**
- **easier implementation of the parallel nonlinearity (an edge effect)**

However, the GK eqn in $(\mu,v_{||})$ coordinates has an additional trapping term which can be difficult to treat numerically due to the discontinuity in the distribution function across the trapped/passing particle boundary.

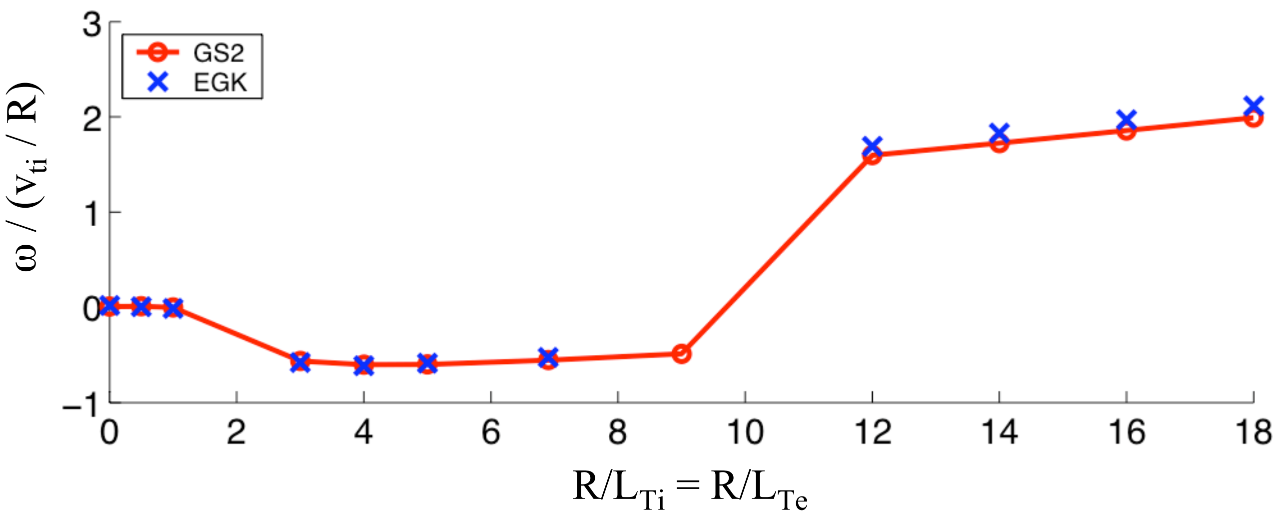
1) CPC **88**, 128 (1995), Phys. Rev. Lett. **85**, 5579 (2000) 2) J. Comput. Phys. **186**, 545 (2003)

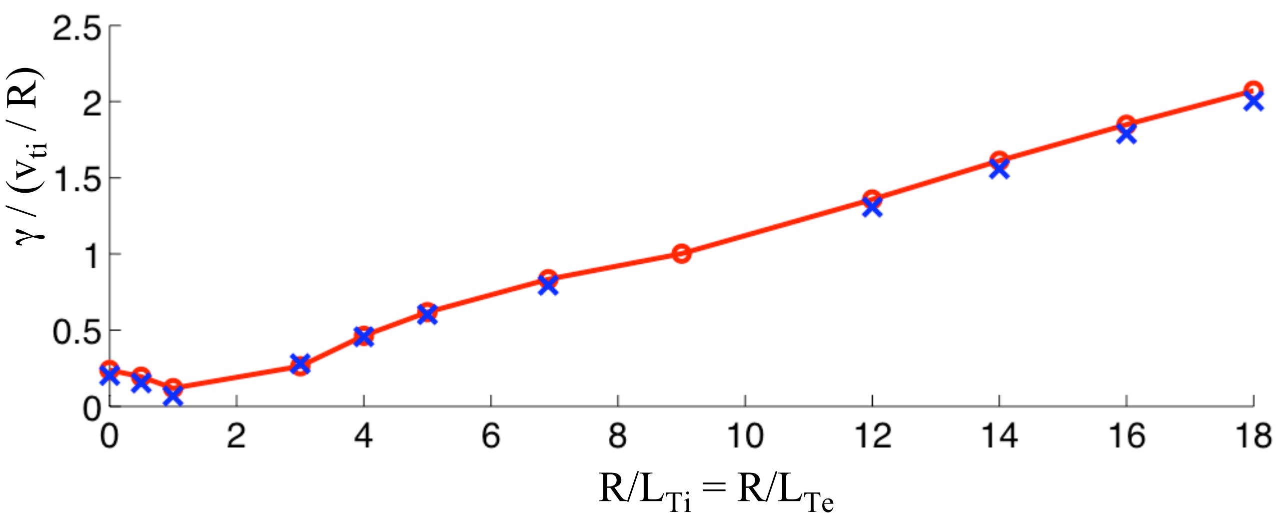
Comparison of EGK with existing $\delta f(R,\mu ,v_{||})$ gyrokinetic codes

	GENE[3] (F. Jenko)	GKV[4] (T. Watanabe & H. Sugama)	EGK
Time integration	3rd order Heun-RK	4th order RK-Gill	3rd order Heun-RK
Phase-Space Derivatives of f	5th & 4th order compact f.d.	5th & 4th order f.d.	3rd order upwinded f.d.
Perpendicular Spatial Dimensions	2D pseudo-spectral (flux tube)	2D pseudo-spectral (flux tube)	Radial grid (periodic or non-periodic b.c.s but no equilibrium radial profile variation), spectral k_y
Physics	Nonlinear, kinetic electrons, electromagnetic	Nonlinear, adiabatic electrons	Linear, kinetic electrons, electrostatic

3) CPC **125**, 196 (2000), CPC **163**, 67 (2004), Plas. Phys. Cont. Fus. **47**, B195 (2005)
4) Nucl. Fus. **46**, 24 (2006)

EGK has been successfully benchmarked with the GS2 code for ITG/TEM linear drift wave physics.



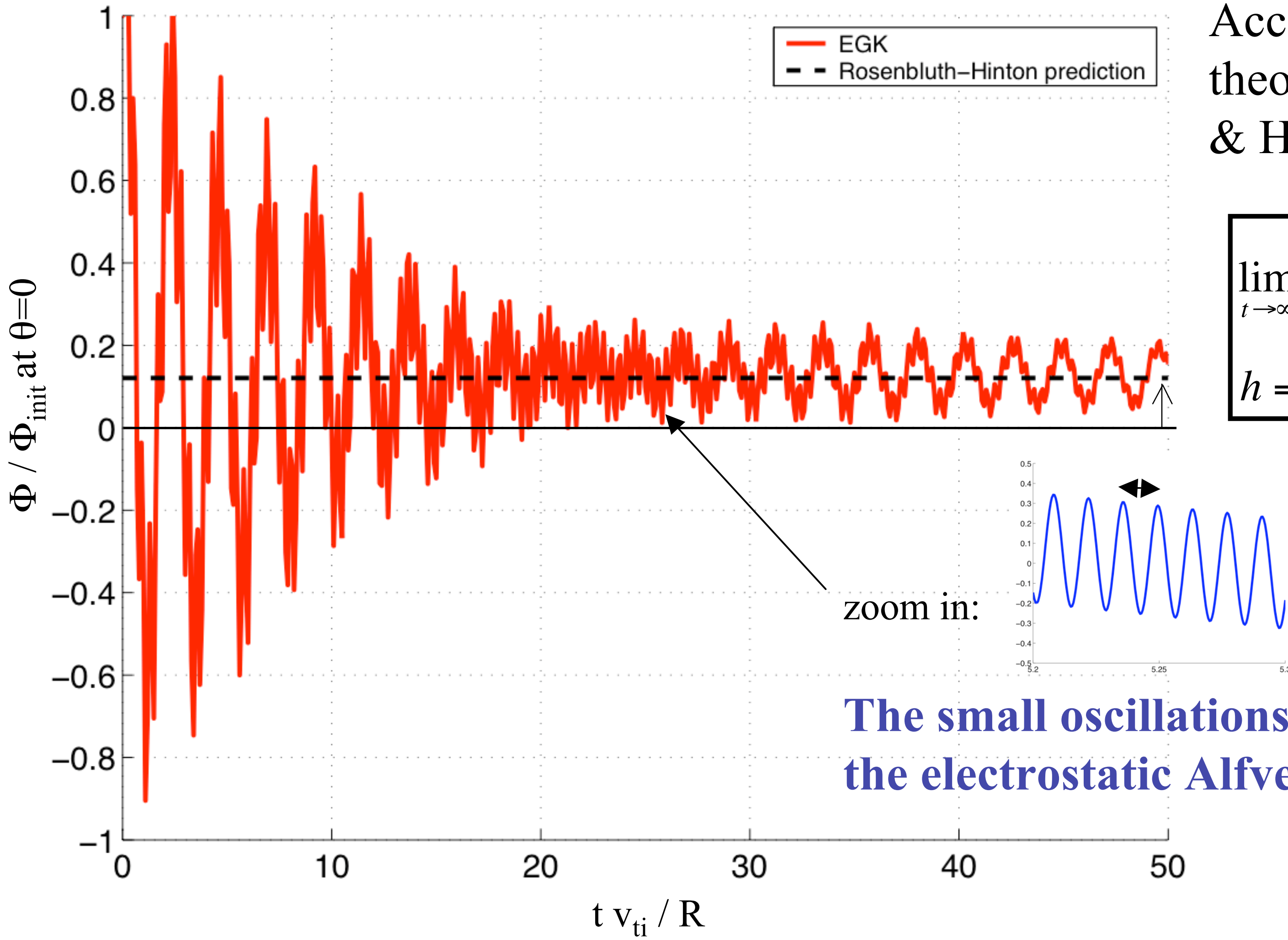


EGK velocity grid:
$\mu/(v_{ts}^2/2B_0) \in [0,25]$
$n\mu = 21$
$v_{\|}/v_{ts} \in [-5,5]$
$nv_{\|} = 41$

GS2: **$n\lambda = 37$, $nE = 16$**

DIII-D Cyclone Base Case Parameters:
s-α geometry
$r/R = 0.18$
$q = 1.388$
$s = 0.8$
$\alpha = 0$
$R/L_{ni} = R/L_{ne} = 2.2$
$T_{0i} = T_{0e}$
$k_y \rho_i = 0.4$

Tests of the collisionless damping of the zonal flow potential were also successful.



According to the theory of Rosenbluth & Hinton[5]:

$$\lim_{t\to\infty}\frac{\Phi}{\Phi_0} = \frac{1}{1+1.6/h}$$
$$h = \sqrt{\varepsilon}/q^2$$

zoom in:

$$T_{Alfven} = 2\pi\frac{k_\perp \rho_s}{k_\| v_{te}}$$

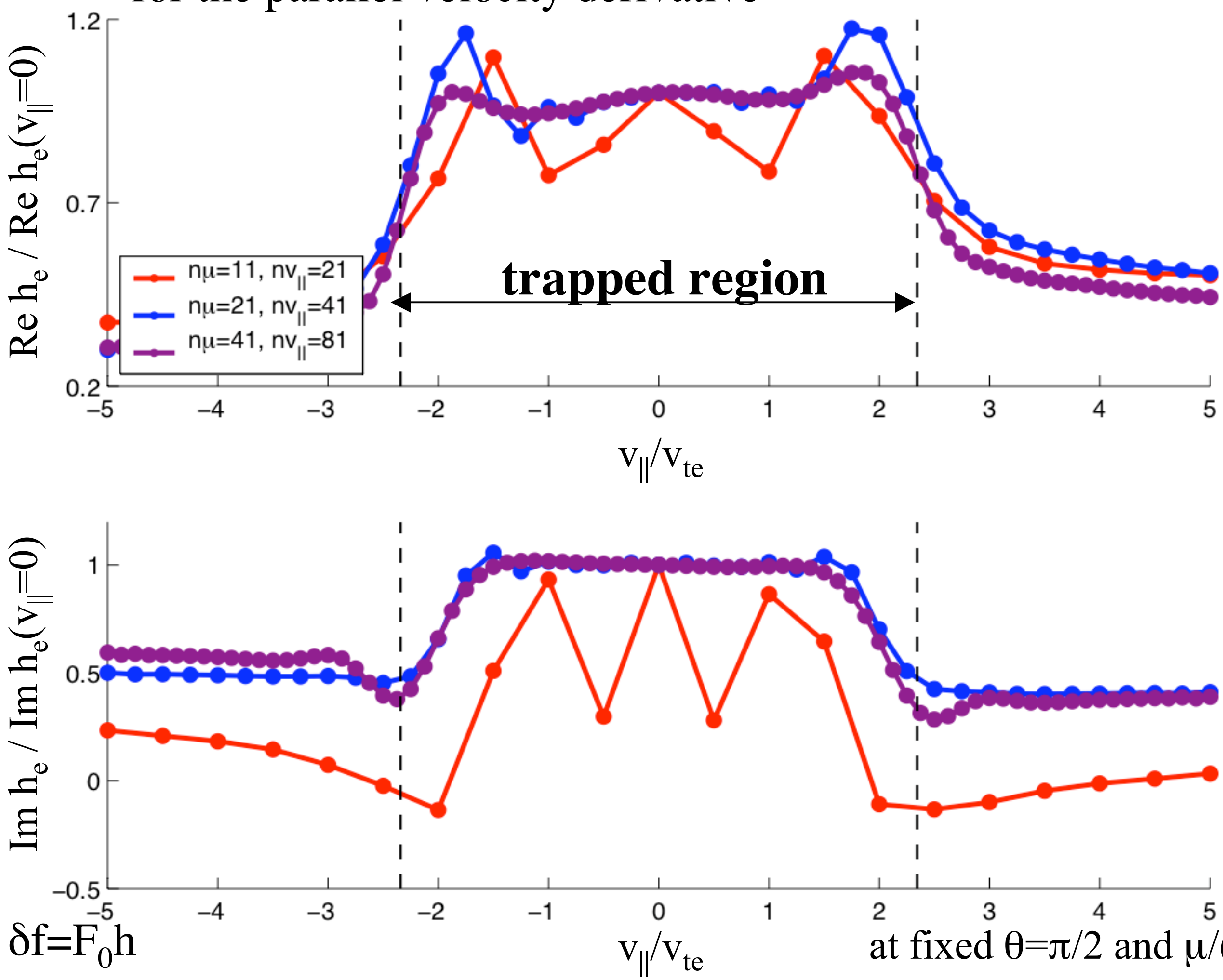
The small oscillations result from the electrostatic Alfven wave.

q = 1.388, r/R = 0.18, $k_x\rho_i$= 0.1

5) Phys. Rev. Lett. **80**, 724 (1998)

EGK has been used to explore various numerical algorithms for the (μ ,$v_{||}$) gyrokinetic formulation.

The behavior of f across the trapped/passing boundary using a 2nd order centered scheme for the parallel velocity derivative



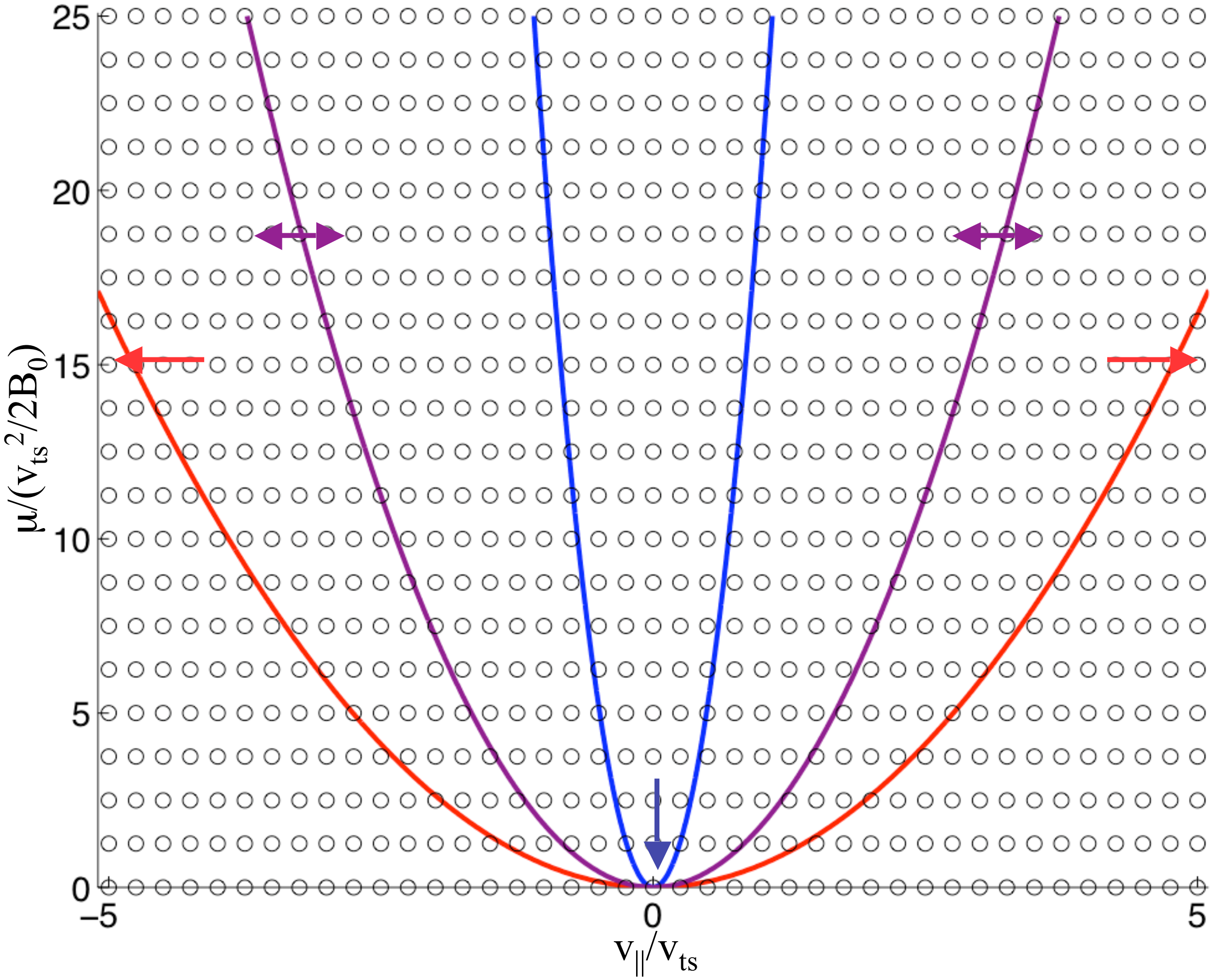
$\delta f = F_0 h$ at fixed $\theta=\pi/2$ and $\mu/(v_{te}^2/2B_0)=25$

With non-dissipative schemes, high velocity space resolution is needed to reduce the numerical Gibbs oscillations which develop in the trapped region and result in inaccuracies in the ITG/TEM growth rates.

A better approach is needed for the treatment of the parallel velocity derivative across the trapped/passing boundary.

Velocity grid for a typical EGK simulation (r/R=0.54)

$$\boxed{v_{\parallel,tp}^2 = 2\mu\left(B_{\max} - B(\theta)\right)}$$

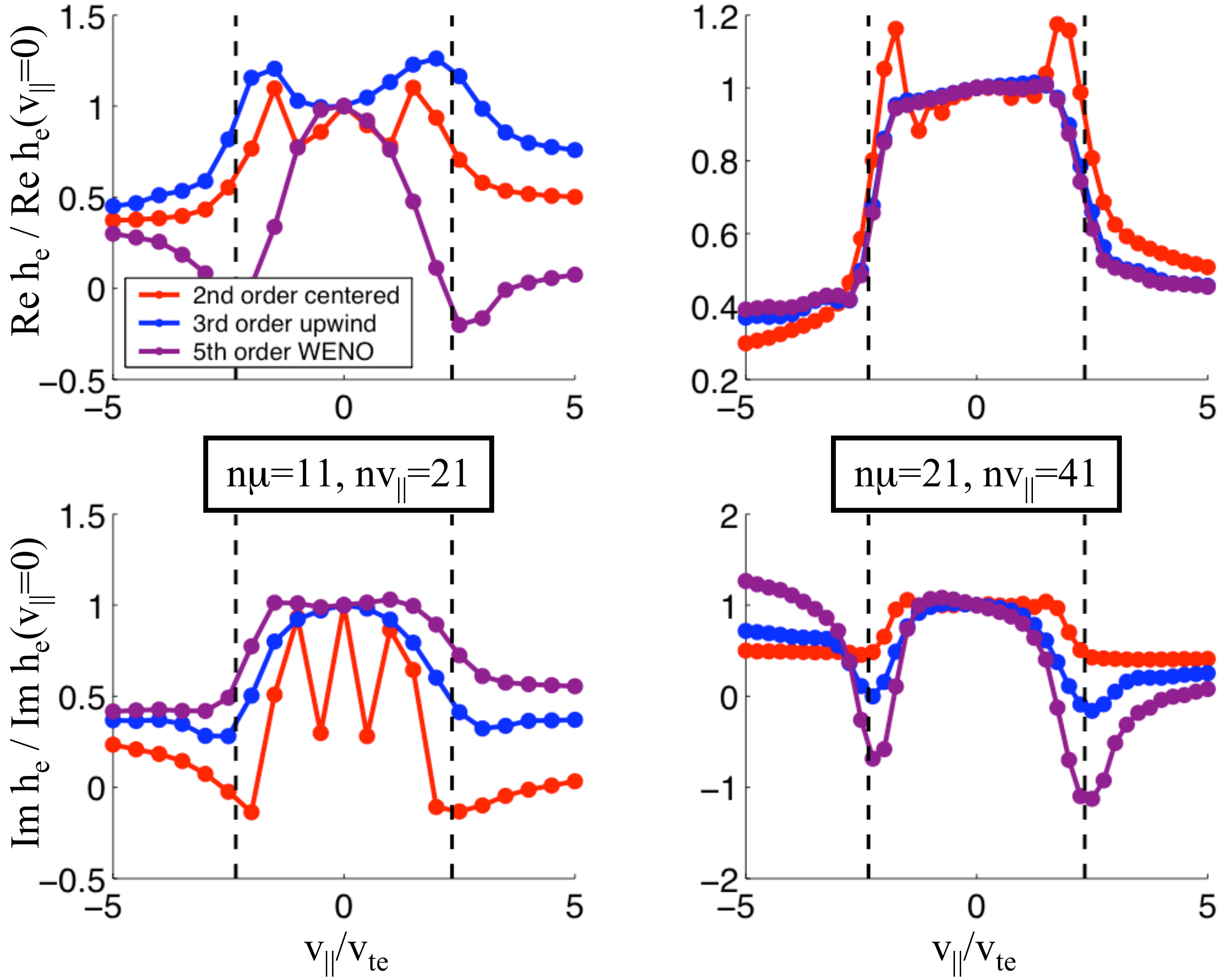


Main Numerical Issues:

1) Stepping over the boundary when computing the derivative
2) One or few points in the trapped region
3) None or few points in the passing region

For now, the best approach is a standard higher-order upwinded finite difference scheme.

Comparison of a centered scheme, an upwinded scheme, and a WENO scheme



The upwinded scheme yields a smoother solution and more accurate growth rate for coarser grid resolution but tends to smear the solution at the discontinuities. The WENO result is similar.

II. Neoclassical Transport Studies

Separate F into a Maxwellian component F_0 + a perturbed component f resulting from magnetic drifts and spatial inhomogeneities.
Steady-state simulation for f.

$$\frac{\partial F_0}{\partial t} + v_{\parallel}\hat{b}\cdot\nabla F_0 - \mu\hat{b}\cdot\nabla B\frac{\partial F_0}{\partial v_{\parallel}} = C(F_0)$$

not solved: $F_0 = F_M$

$$\frac{\partial f}{\partial t} + \left(v_{\parallel}\hat{b}\cdot\nabla + \vec{v}_D\cdot\nabla - \mu\hat{b}\cdot\nabla B\frac{\partial}{\partial v_{\parallel}}\right)f$$

$$= ZF_0\left(-v_{\parallel}\hat{b}\cdot\nabla - \vec{v}_D\cdot\nabla\right)\frac{e\Phi}{T_0}$$

$$\boxed{-\vec{v}_D\cdot\nabla F_0} + C(f)$$

Add the neoclassical driving term to the δf kinetic eqn

$$\sum_s \frac{Z_s^2 e^2 n_{0s}}{T_{0s}}(1-\Gamma_{0s})\Phi = \sum_s Z_s e\int d^3v f_s$$

This builds on the work of Wang et al.[6] and Satake et al.[7] but includes kinetic electrons and the poloidal variation of Φ.

6) Phys. Plasmas **13**, 082501(2006)

7) Nucl. Fus. **45**, 1362 (2005)

Initial studies including the neoclassical self-consistent E_r assume $\Phi \neq fnc(\theta)$.

$$\Phi(r,\theta) = -rE_r^0 + \cancel{\delta\Phi(r,\theta)}$$

$$\frac{\partial f}{\partial t} + \left(v_{\parallel}\hat{b}\cdot\nabla + \cancel{\vec{v}_D\cdot\nabla} - \mu\hat{b}\cdot\nabla B\frac{\partial}{\partial v_{\parallel}} \right) f$$

$$= ZF_0\left(-v_{\parallel}\hat{b}\cdot\nabla - \vec{v}_D\cdot\nabla\right)\cancel{\frac{e\delta\Phi}{T_0}}$$

$$-\vec{v}_D\cdot\nabla F_0 - \vec{v}_D\cdot\nabla\Phi^0 + C(f)$$

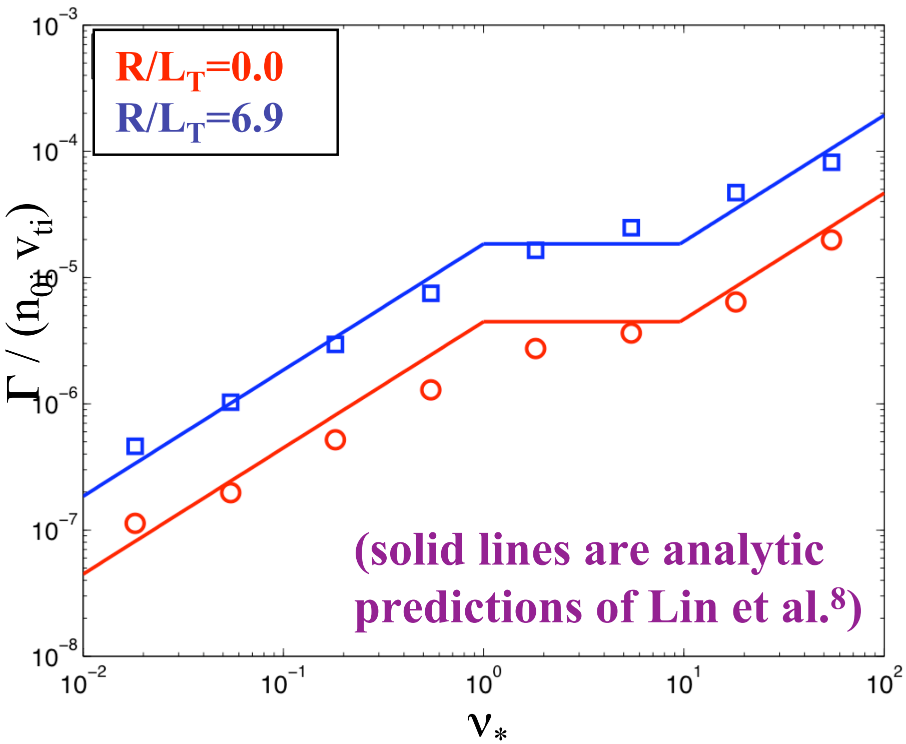
This is a radially local problem using a fixed constant value for the equilibrium density and temperature gradients. It requires no Poisson equation coupling.

This problem was solved by Wang et al. & Satake et al. (retaining the FOW term and using a radially global simulation) via coulping the kinetic eqn with the vorticity constraint eqn $\frac{n_{0i}m_i}{\langle B^2\rangle}\frac{\partial^2\Phi^0}{\partial t\partial r} = Z_i e\int d^3v f_i \vec{v}_D\cdot\nabla r$ to compute a saturated E_r^0 which corresponds to zero parallel ion flow.

However, more generally, it can be solved as above without the vorticity constraint eqn coupling and can be viewed as the solution of the saturated f (and associated moments like $u_{\parallel}$) for a given equilibrium E_r^0.

Using pitch angle scattering collisions, our local code has been successfully benchmarked.

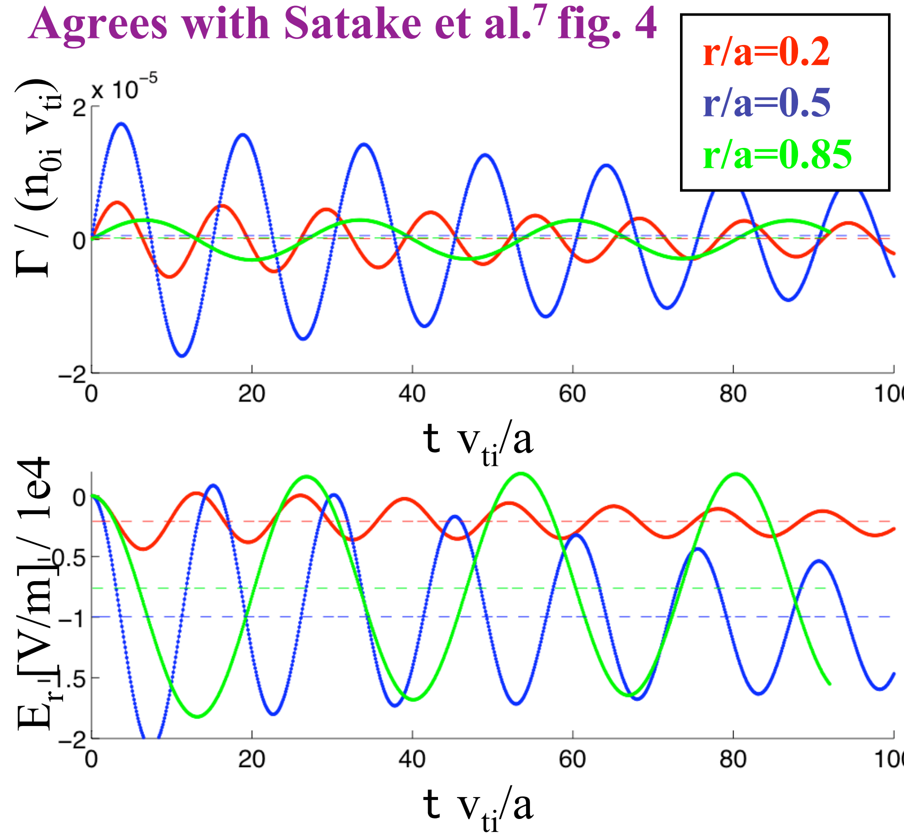
$E_r^0=0$



$r/R=0.18$ $\rho/R=0.001$
$q=1.388$ $R/L_n=2.2$

Including E_r^0 (vorticity eqn coupling)

Agrees with Satake et al.[7] fig. 4



$R/a=4.0$ $\rho/a=0.0039$ n, T vary
$q=3.0$ $\nu/v_{ti}/a=0.1$ R/L_n, R/L_T vary

8) Phys. Plasmas **2**, 2975 (1995)

These neoclassical studies have recently been extended to include the poloidal variation of Φ.

$$\Phi(r,\theta) = -rE_r^0 + \delta\Phi(\cancel{r},\theta)$$

$$\frac{\partial f}{\partial t} + \left(v_\parallel \hat{b}\cdot\nabla + \cancel{\vec{v}_D\cdot\nabla} - \mu\hat{b}\cdot\nabla B\,\frac{\partial}{\partial v_\parallel}\right) f$$

$$= ZF_0\left(-v_\parallel\hat{b}\cdot\nabla - \cancel{\vec{v}_D\cdot\nabla}\right)\frac{e\delta\Phi}{T_0}$$

$$-\vec{v}_D\cdot\nabla F_0 - \vec{v}_D\cdot\nabla\Phi^0 + C(f)$$

$$\sum_s \frac{Z_s^2 e^2 n_{0s}}{T_{0s}}\cancel{(1-\Gamma_{0s})}\Phi = \sum_s Z_s e\int d^3v f_s$$

For adiabatic electrons: $$\frac{e\delta\Phi}{T_{0e}} = Z_i\int d^3v f_i$$

$$(\langle\delta\Phi\rangle = 0)$$

Next order (poloidal) corrections are still solved as a radially local problem but now involve Poisson eqn coupling.

In agreement with analytical theory, we find that δΦ varies sinusoidally and depends on the collisionality.

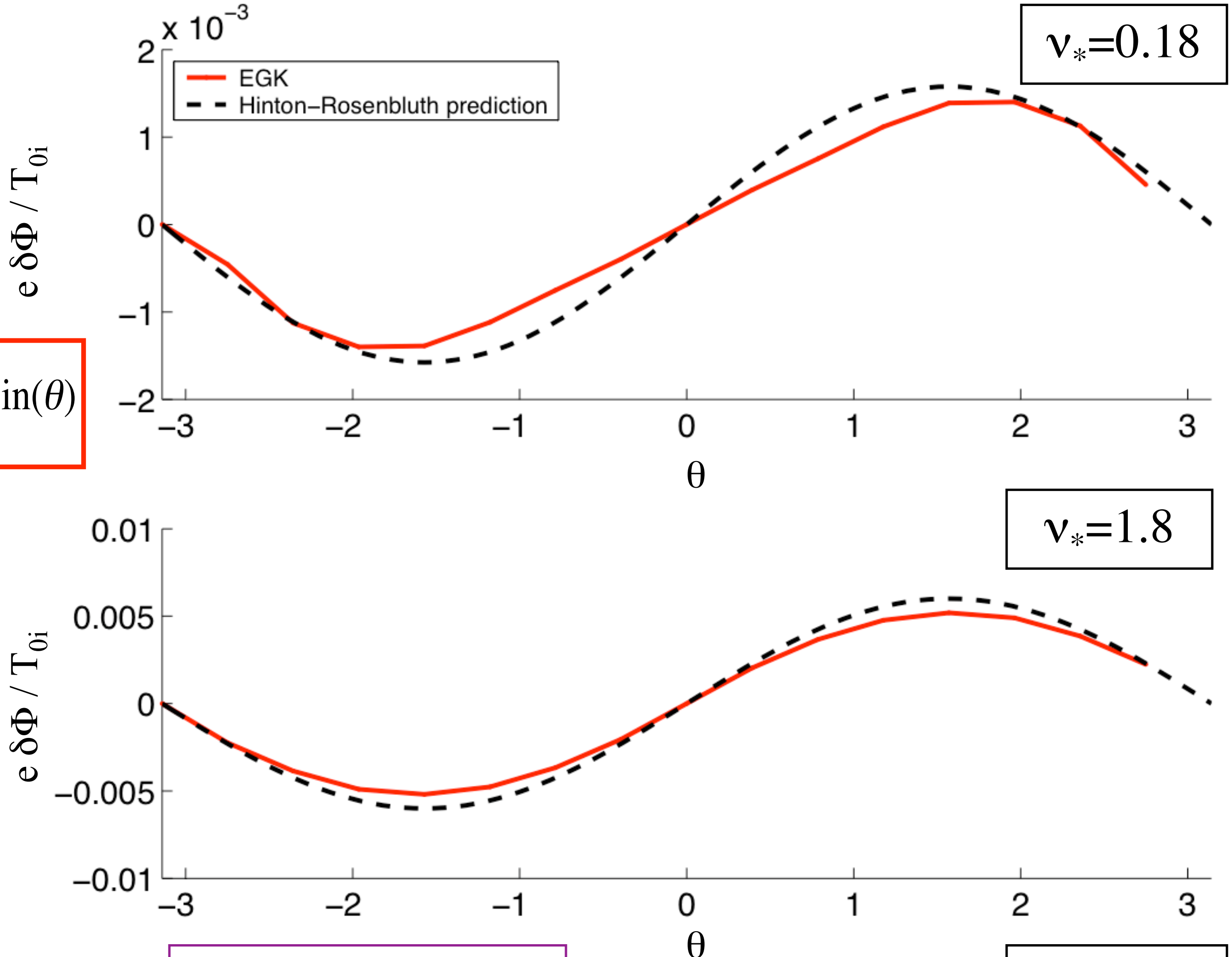
Hinton & Rosenbluth analytical result[9]:

Banana regime

$$\boxed{\frac{e\delta\Phi}{T_{oi}} \sim \frac{1.81}{\varepsilon^{3/2}(1+\tau)} q^2 \frac{\rho_o}{L_T} \frac{\nu_{ii}}{v_{ti}/R} \sin(\theta)}$$

Plateau regime

$$\boxed{\frac{e\delta\Phi}{T_{oi}} \sim \frac{\sqrt{2\pi}}{2(1+\tau)} q \frac{\rho_o}{L_T} \sin(\theta)}$$



r/R=0.18 ρ/R=0.001
q=1.388 R/L_n=2.2

E_r^0 = 0.0
R/L_T = 6.9

9) Phys. Fluids **16**, 836 (1973)

Unlike the lowest order problem, the radial particle flux with poloidal variation included is not exactly zero.

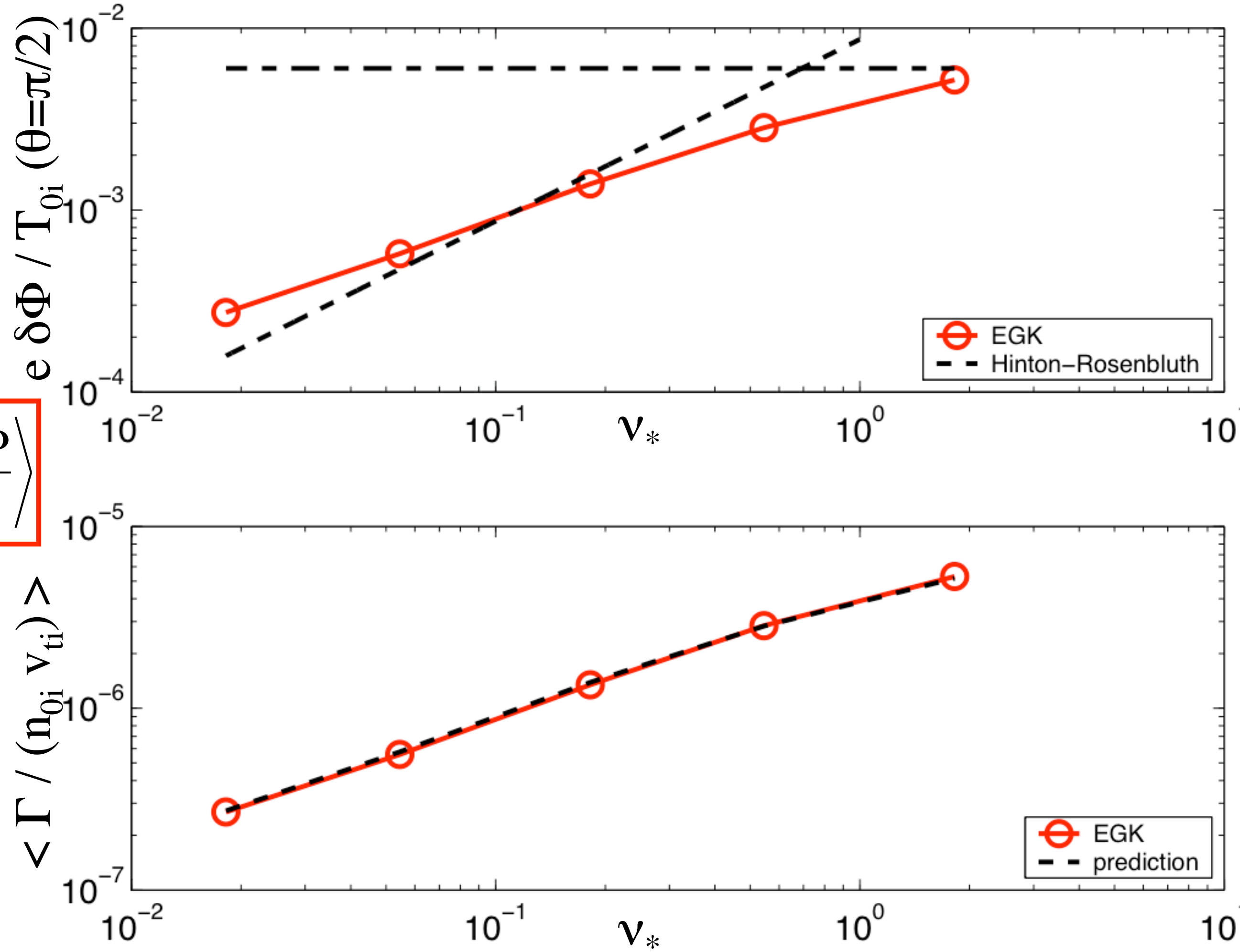
Operating on kinetic eqn with $\left\langle \int d^3v \frac{v_\parallel}{B} \cdots \right\rangle$ yields (in steady state):

$$\boxed{\left\langle \frac{\Gamma}{n_{0i} v_{ti}} \right\rangle = \frac{1}{\varepsilon} \frac{\rho_0}{R} \frac{e}{T_{oi}} \left\langle \frac{B_0}{B} \frac{\partial \delta\Phi}{\partial \theta} \right\rangle}$$

Using that

$\delta\Phi(\theta)=C\sin(\theta)$:

$$\left\langle \frac{\Gamma}{n_{0i} v_{ti}} \right\rangle = \frac{\rho_0}{R} \frac{e}{T_{oi}} C$$



$E_r^0 = 0.0$
$R/L_T = 6.9$

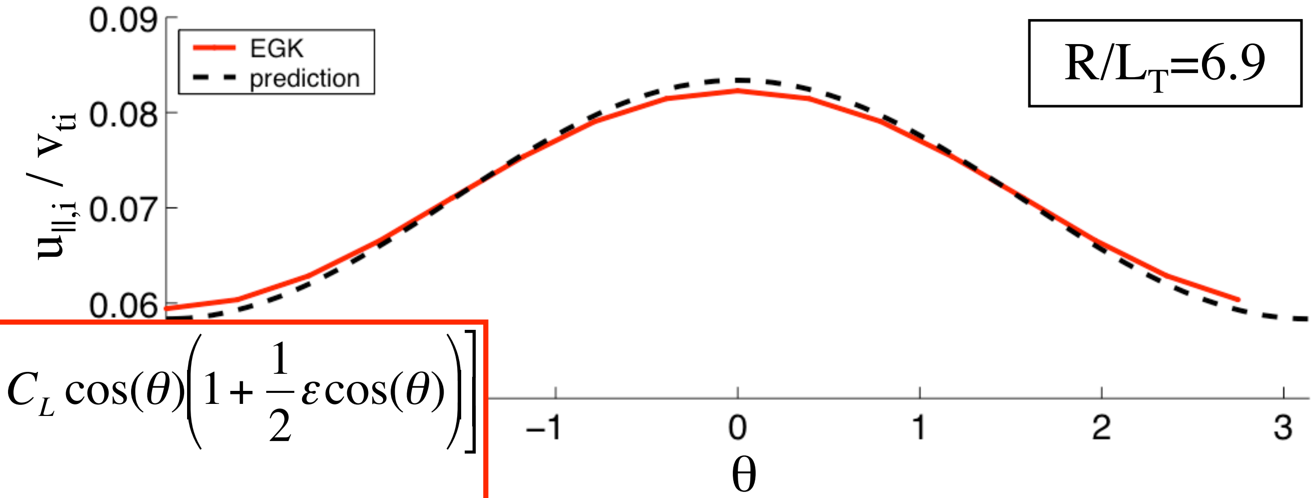
The resulting effect on the poloidal variation of the ion parallel ion flow has also been studied.

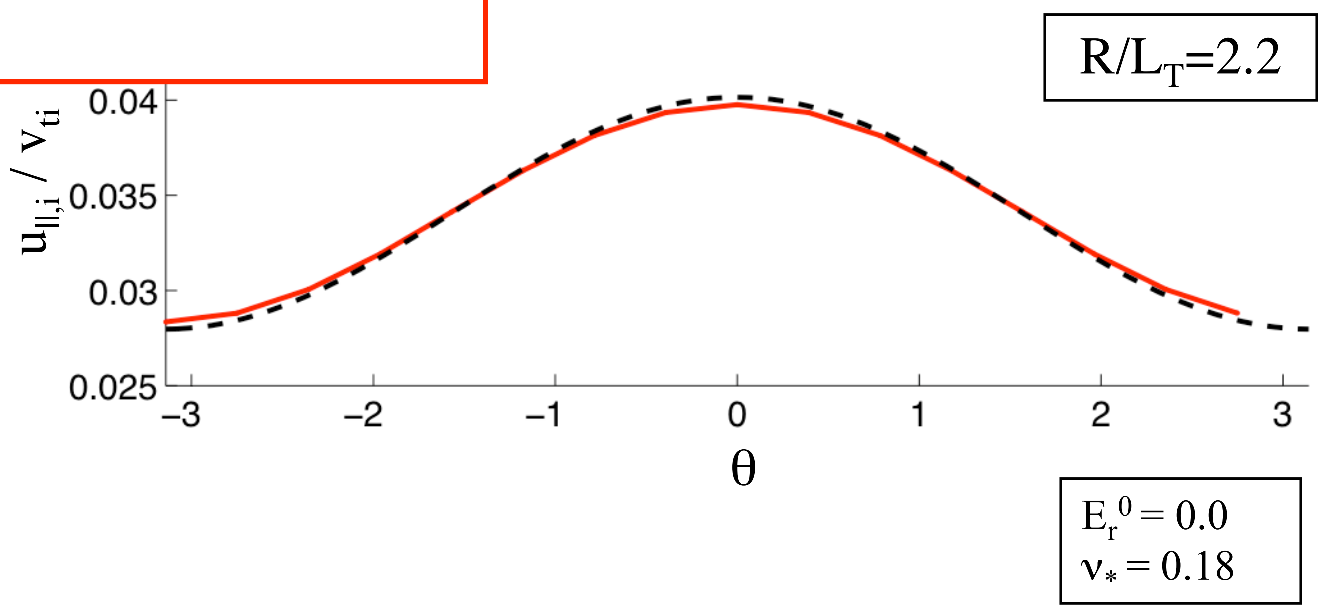
Operating on the kinetic eqn with $\int d^3v$ yields (in steady-state):

$$u_{\|}(\theta) = \frac{1}{1+\varepsilon\cos(\theta)}\left[const + C_L\cos(\theta)\left(1+\frac{1}{2}\varepsilon\cos(\theta)\right)\right]$$

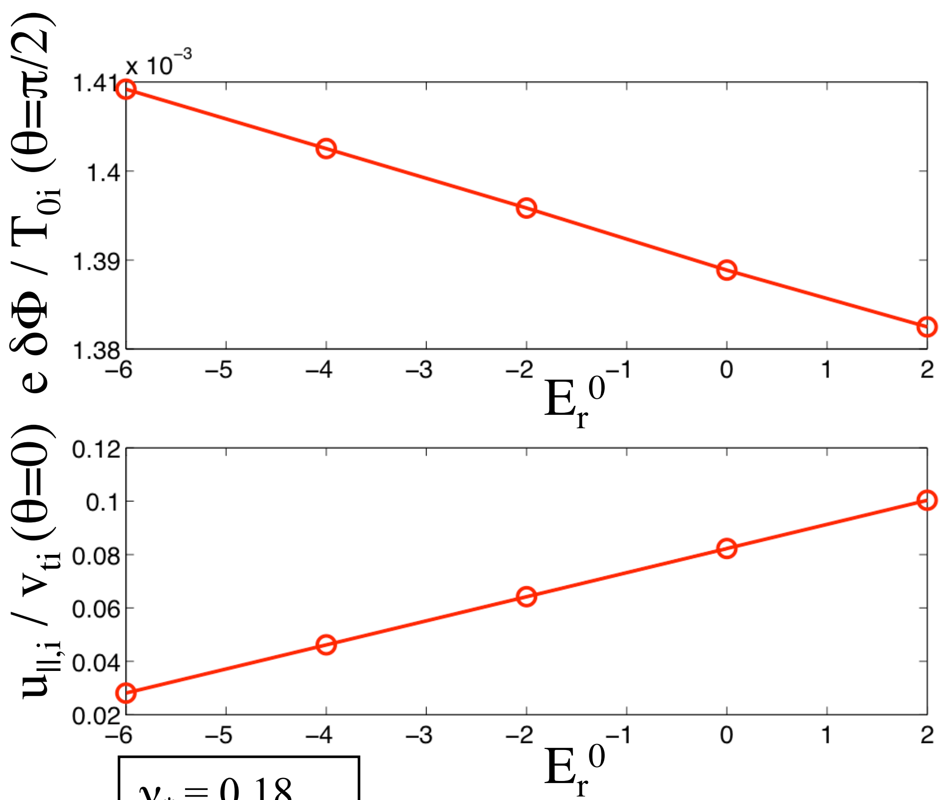
$$C_L \equiv 2q\left(\frac{\rho_0}{L_n} + \frac{\rho_0}{L_T} + \frac{Ze}{T_{0i}}E_r^0\right)$$

linear dependence on the equilibrium gradients

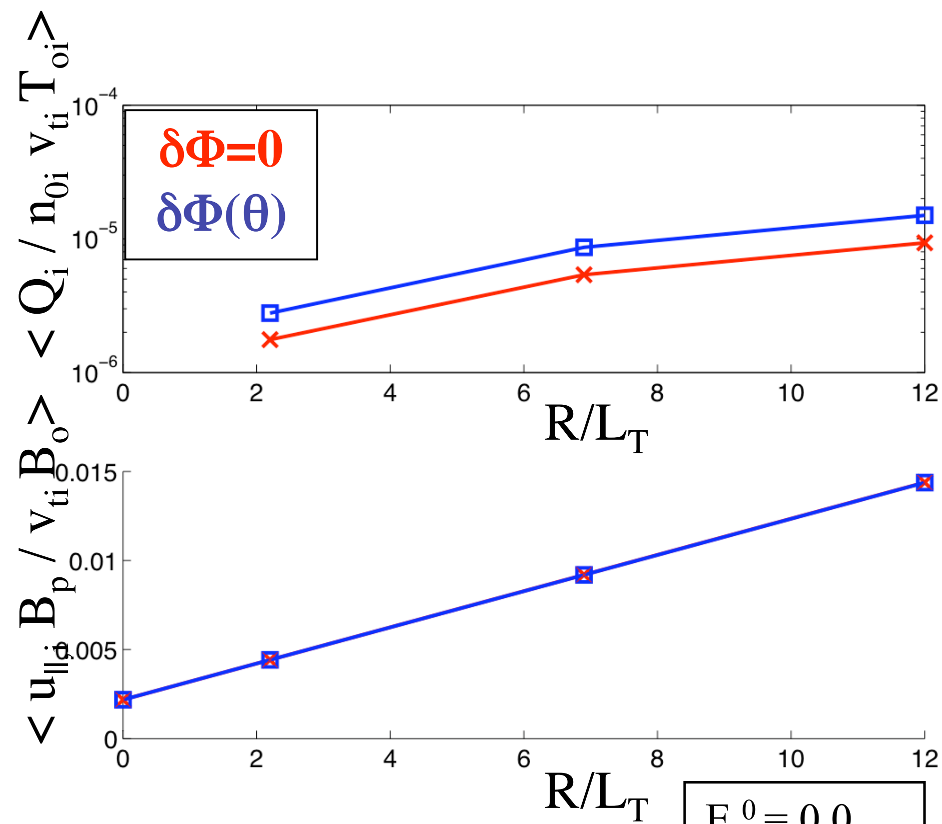




Overall, we find that the poloidal variation of the potential is weak, as expected, as are its effects.



$\nu_* = 0.18$
$R/L_T = 6.9$



$E_r^0 = 0.0$
$\nu_* = 0.18$

The total potential scales as:
$\Phi \sim -rE_r^0\,(1+h)$, where $h \ll 1$

The effects of kinetic electron dynamics are also being studied.

The k_x=0 Poisson eqn becomes:

(i.e. no explicit Φ dependence)

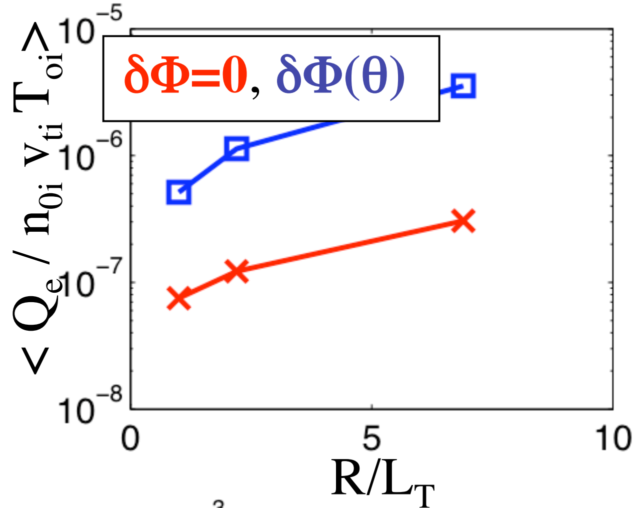
$$\boxed{0 = \sum_s Z_s \int d^3 v f_s}$$

Thus, numerical solution of the kinetic+Poisson equations requires an implicit algorithm.

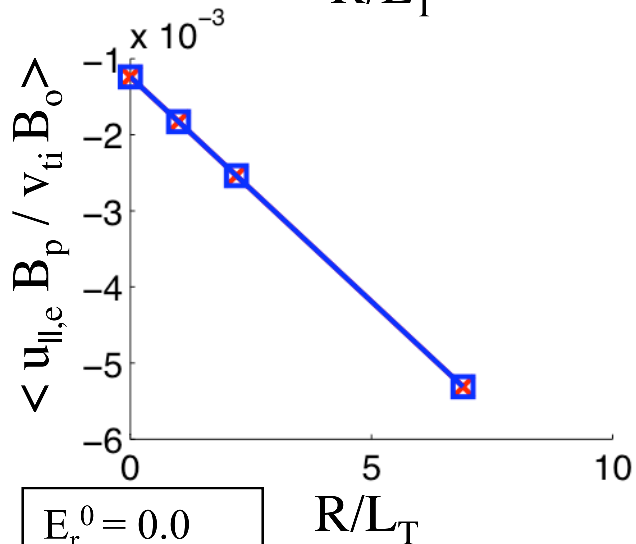
We use a semi-implicit algorithm in which the parallel free-streaming dynamics are treated implicitly, while all other dynamics are treated explicitly.

Note that our previous prediction of the steady-state radial particle flux holds independently for both ion and electron species, so the plasma will be ambipolar.

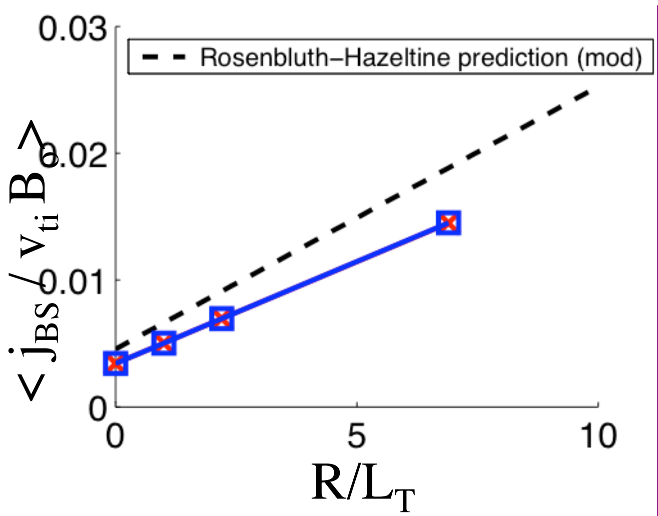
Preliminary results indicate that, while the poloidal variation of the potential does not significantly affect the ion dynamics, it does produce an enhanced electron heat flux.



This effect on Q qualitatively agrees with the analytical results of Stringer et al.[10] The resulting bootstrap also qualitatively agrees with the Rosenbluth-Hazeltine prediction[11].



$E_r^{\,0} = 0.0$
$\nu_* = 0.18$



A more realistic collision operator is needed for direct comparisons with analytic neoclassical theory. This will be explored next. Here we have modified the R-H coeff for the temp. gradient to be the same as for the density gradient.

10) Phys. Fluids B **3**, 981 (1991)
11) Phys. Fluids **15**, 116 (1972)

The next step is to add finite k_x effects.

$$\Phi(r,\theta) = -rE_r^0 + \delta\Phi(r,\theta)$$

$$\frac{\partial f}{\partial t} + \left(v_{\parallel}\hat{b}\cdot\nabla + \vec{v}_D\cdot\nabla - \mu\hat{b}\cdot\nabla B\frac{\partial}{\partial v_{\parallel}} \right) f$$

$$= ZF_0\left(-v_{\parallel}\hat{b}\cdot\nabla - \vec{v}_D\cdot\nabla\right)\frac{e\delta\Phi}{T_0}$$

$$-\vec{v}_D\cdot\nabla F_0 - \vec{v}_D\cdot\nabla\Phi^0 + C(f)$$

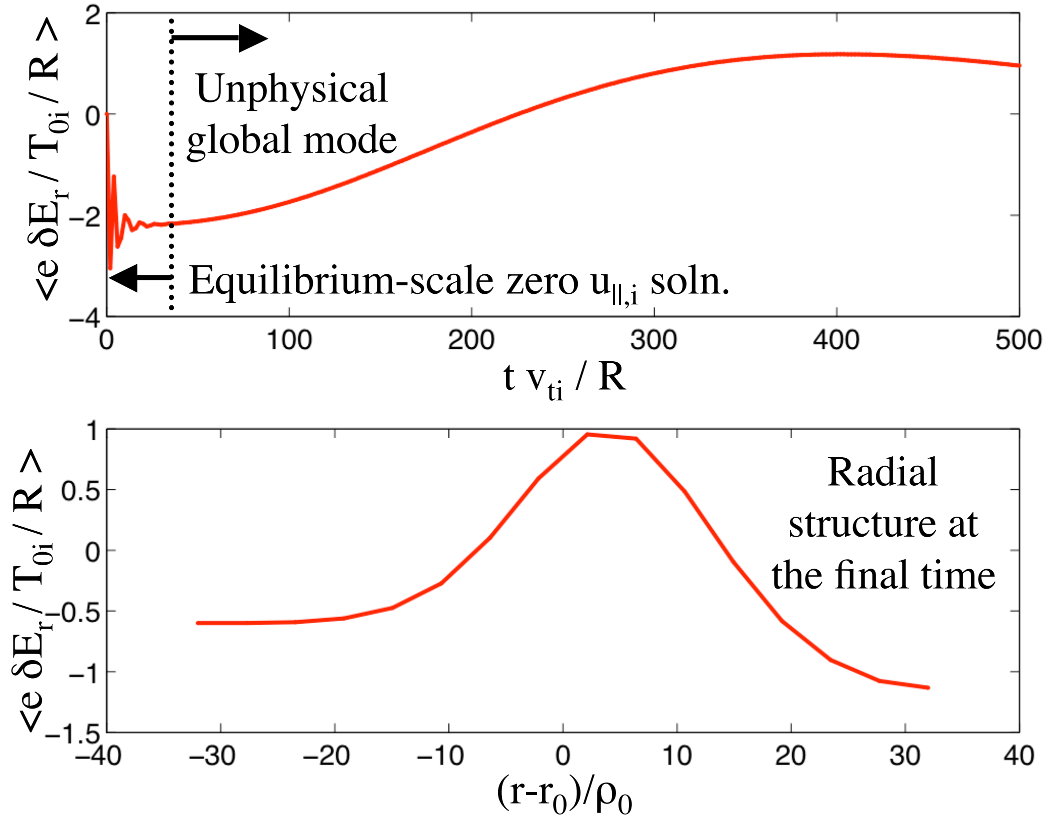
$$\sum_s \frac{Z_s^2 e^2 n_{0s}}{T_{0s}}\left(1-\Gamma_{0s}\right)\Phi = \sum_s Z_s e\int d^3v f_s$$

Can the Poisson eqn for the neoclassical problem be solved using the same algorithms as for drift waves?

This includes the effects of the finite orbit width term, which Wang et al.[6] showed can be significant for NSTX plasmas. This problem requires a global simulation with equilibrium profile variation.

As we move toward a unified drift wave + neoclassical transport code, a primary issue will be the radial b.c.s for Φ.

Drift wave turbulence generally uses zero radial boundary conditions. However, for the neoclassical problem, we find that the choice of the radial b.c.s can induce unphysical global modes on the long time scale.



Here we write the LHS operator of the Poisson eqn as $-\partial^2\Phi/\partial r^2$ and enforce the conditions $\sum\Phi=0$, $\sum r\Phi=0$. The FOW term is neglected and we use a set of radially local equilibrium parameters. This should produce a small correction to the equilibrium E_r^0 which is approximately radially constant. However, instead, the equilibrium E_r^0 corresponding to zero parallel ion flow is regenerated at small time scales and then an unphysical global mode with a definite radial structure develops. The level of the saturated E_r for this mode is found to be determined by the b.c.s.

This implies that a different approach is needed for solving the Poisson eqn for the neoclassical potential.

Summary of EGK Results

Linear Gyrokinetics:

EGK solves the linear $\delta f\,(r,\theta,k_y,\mu,v_{||})$ GK eqns in the electrostatic limit. It includes gyrokinetic electrons and trapped particle dynamics.

- **Successful benchmarks of ITG/TEM linear drift wave physics & collisionless damping of zonal flows have been completed.**
- **Velocity space dissipation algorithms for $(\mu,v_{||})$ have been explored. We find that a careful numerical treatment of the trapping term is needed.**

Neoclassical Transport:

EGK solves the linear δf drift kinetic with the neoclassical driver using the radially local limit and a pitch angle scattering collision operator.

- **Successful benchmarks of E_r neglecting the poloidal variation of Φ and coupling with the vortivity constraint eqn. have been completed.**
- **New extended studies including the poloidal variation of Φ and kinetic electron dynamics indicate that the poloidal variation is weak and does not significantly affect the ion dynamics, but does enhance the electron heat flux. This agrees qualitatively with analytic neoclassical theory.**

Next Studies: Unified Simulations of Drift Wave Turbulence and Neoclassical Transport