TEMPEST Simulations of the Radial Electric Field Dynamics in the Neoclassical Plasmas

Z. Xiong, X. Q. Xu, W. M. Nevins, R. H. Cohen, and ESL Team

Lawrence Livermore National Laboratory, Livermore, CA 94550 USA,

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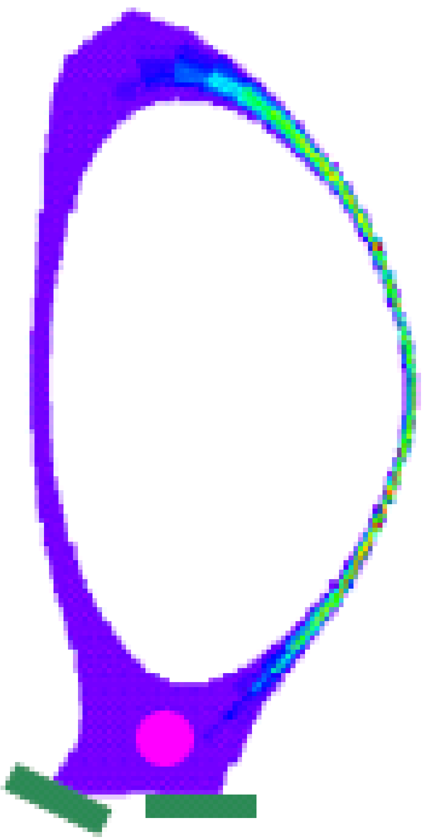
Principal Results

- **TEMPEST is a fully nonlinear (full-f) five dimensional (3d2v) gyrokinetic continuum edge-plasma code.**
- **The four-dimensional (2d2v) version of the code correctly produces frequency, collisionless damping of geodesic acoustic modes and zonal flow (Rosenbluth-Hinton residual) with Boltzmann electrons using a full-f code in small ε limit**
- **The electric field via Gyrokinetic Poisson Eq is also found to agree with the standard neoclassical expression for steep density and ion temperature gradients in the plateau regime with Boltzmann electrons**
- **The preliminary encouraging results demonstrates the emerging capability of the TEMPEST code.**

TEMPEST, a fully nonlinear (full-f) gyrokinetic continuum code

- **5D (ψ,θ,ζ,E_0, μ) continuum code; part of Edge Simulation Laboratory(ESL) project**
- **Realistic X-point divertor geometry**
  - **Open + closed flux surfaces**
- **an implicit backward-differencing scheme in time**
  - **using a Newton-Krylov iteration**
- **Higher order accuracy in phase space:**
  - **spatial derivatives: finite differences**
    - **Fourth order upwinding & Weno scheme**
  - **finite volume method is used in velocity space (E_0, μ) for FP collision**
- **Uses *Hypre* library of parallel linear algebra solvers and preconditioners**



Fully nonlinear ion gyrokinetic equation has been cast in E_0-μ coordinates

$$
\begin{aligned}
\frac{\partial F_\alpha}{\partial t} &+ \bar{\mathrm{v}}_{\mathrm{d}} \cdot \frac{\partial F_\alpha}{\partial \bar{\mathrm{x}}_\perp} + (\bar{v}_{\|\alpha} + v_{Banos})\mathrm{b} \cdot \frac{\partial F_\alpha}{\partial \bar{\mathrm{x}}} \\
&+ \left[q\frac{\partial \langle \Phi^0 \rangle}{\partial t} + \bar{\mu}\frac{\partial B}{\partial t} - \frac{B}{B^*}\bar{v}_\| q \frac{\partial \langle \delta\phi \rangle}{\partial s} - \mathrm{v}_{\mathrm{d}}^{\mathbf{0}} \cdot (q\bar{\nabla}\langle \delta\phi \rangle) \right] \frac{\partial F_\alpha}{\partial E_0} \\
&= C(F_\alpha, F_\alpha), \\
\bar{\mathrm{v}}_{\mathrm{d}} &= \frac{c\mathrm{b}}{qB_\|^*} \times \left(q\bar{\nabla}\langle \Phi \rangle + \bar{\mu}\bar{\nabla}B \right) + \bar{v}_\|^2 \frac{M_\alpha c}{qB_\|^*}(\bar{\nabla}\times\mathrm{b}). \\
\bar{\mathrm{v}}_{\mathrm{d}}^{\mathbf{0}} &= \frac{c\mathrm{b}}{qB_\|^*} \times \left(q\bar{\nabla}\langle \Phi^0 \rangle + \bar{\mu}\bar{\nabla}B \right) + \bar{v}_\|^2 \frac{M_\alpha c}{qB_\|^*}(\bar{\nabla}\times\mathrm{b}). \\
\bar{v}_\| &= \pm\sqrt{\frac{2}{M_\alpha}\left(E_0 - \bar{\mu}B - q\langle \Phi^0 \rangle\right)}, \\
v_{Banos} &= \frac{\mu c}{q}(\mathrm{b}\cdot\bar{\nabla}\times\mathrm{b}), \\
B_{\|\alpha}^* &\equiv B\left[1 + \frac{\mathrm{b}}{\Omega_{c\alpha}} \cdot (v_\| \bar{\nabla}\times\mathrm{b})\right], \Omega_{c\alpha} = \frac{qB}{M_\alpha c}, \mu = \frac{\mathrm{M}_\alpha \mathrm{v}_\perp^2}{2\mathrm{B}}, \\
\langle \delta\phi \rangle &= \langle \Phi \rangle - \langle \Phi^0 \rangle.
\end{aligned}
$$

Qin, Cohen, Nevins, and Xu,

Contrib. Plasma Phys. 46, 7-9, 477 (2006)

- ✓**The field is split into two parts: Φ^0 and $\delta\phi$;**
- ✓**$E_0=mv^2/2+q\Phi^0$, a constant of motion if $\delta\phi\sim0$ and a coordinate aligned with flow;**
- ✓**E_0xB flow terms and the slow variation Φ^0 from Qin's formulation will be added.**

Fully nonlinear gyro-kinetic Poisson equation in the long wavelength limit

In the long wavelength limit $k_\perp \rho_\alpha \ll 1$, the self-consistent electromagnetic field are typically computed from the gyro-kinetic Poisson equation for the multiple species

$$\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2} \frac{1}{N_\alpha} (\nabla_\perp \cdot N_\alpha \nabla_\perp \Phi) + \nabla^2 \Phi = -4\pi e \left[\sum_\alpha Z_\alpha N_\alpha(\mathbf{x}, t) - n_e(\mathbf{x}, t) \right] - \sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2} \frac{1}{N_\alpha Z_\alpha e} \nabla_\perp^2 p_{\perp\alpha}.$$

where the gyrocenter center density N_α and perpendicular ion pressure $p_{\perp\alpha}$ are defined by

$$\begin{aligned}
N_\alpha(\mathbf{x}, t) &\equiv \frac{2\pi}{M_\alpha} \int B_\parallel^* d\bar{v}_\parallel d\bar{\mu} F_\alpha, \\
n_e(\mathbf{x}, t) &\equiv \frac{2\pi}{m_e} \int B_\parallel^* dv_\parallel d\mu f_e, \\
p_{\perp\alpha} &= \pi B \int dv_\parallel d\bar{\mu} (v_\perp^2 F_\alpha), \\
T_{\perp\alpha} &= \frac{p_{\perp\alpha}}{N_\alpha(\mathbf{x}, t)}.
\end{aligned}$$

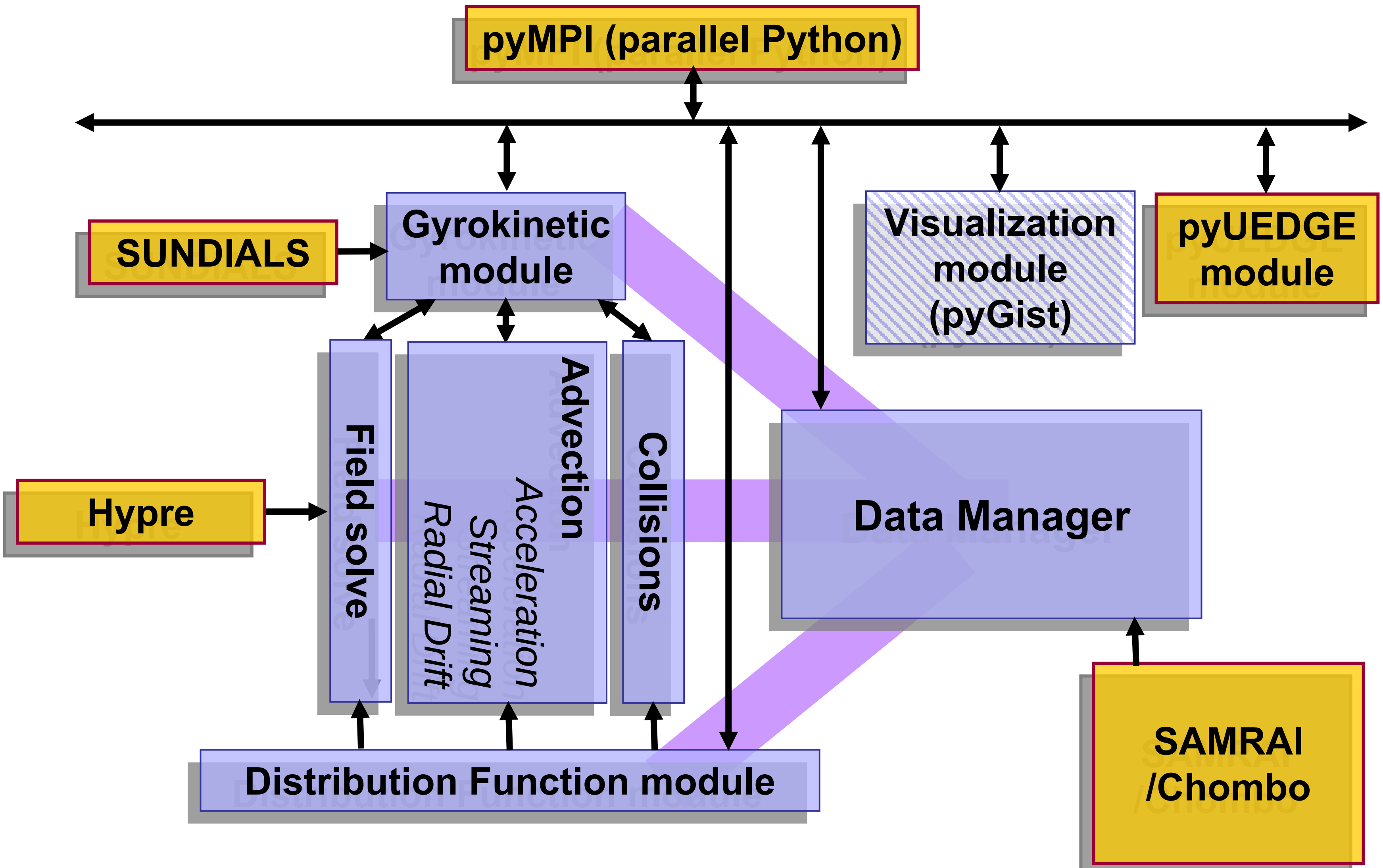
The n_α and $T_{\perp\alpha}$ are the normalization density and temperature. The ion gyroradius is $\rho_\alpha = \sqrt{2T_{\perp\alpha}/M_\alpha}/\Omega_\alpha$, the ion gyrofrequency is $\Omega_\alpha = Z_\alpha eB/M_\alpha c$, and the ion Debye length is $\lambda_{D\alpha}^2 = T_{\perp\alpha}/4\pi n_\alpha Z_\alpha^2 e^2$.

✓**Diamagnetic density is included;**

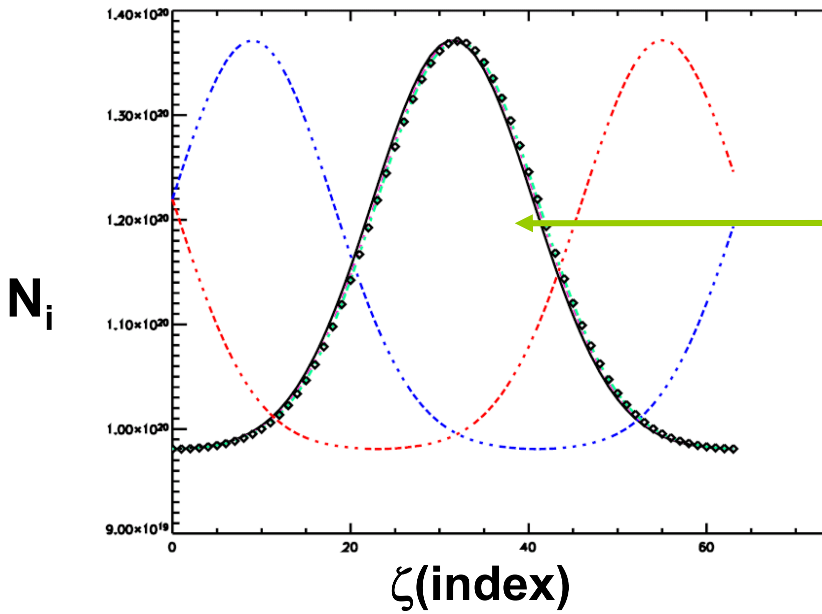
✓**It is fully nonlinear since the N_α and P_α are calculated from F_α;**

✓**If the first-order Pade approximation to $\Gamma_0 = 1/(1 + b)$ for the modified Bessel function is used, then the same field solve will be used in the arbitrary wavelength regime.**

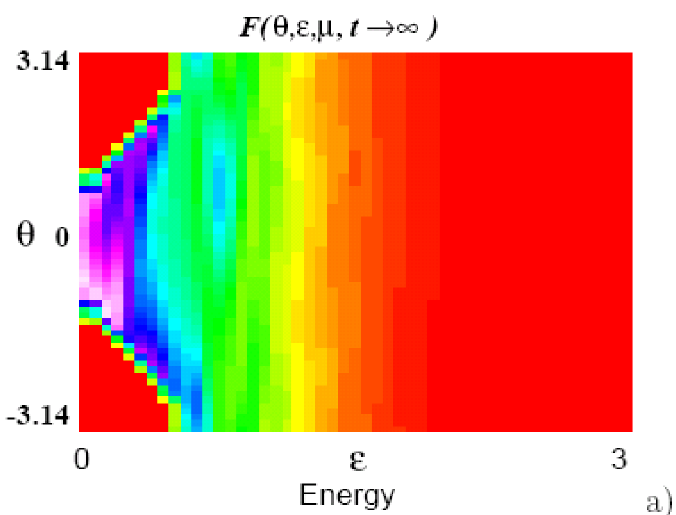
We have designed and implemented a 5D edge simulation framework

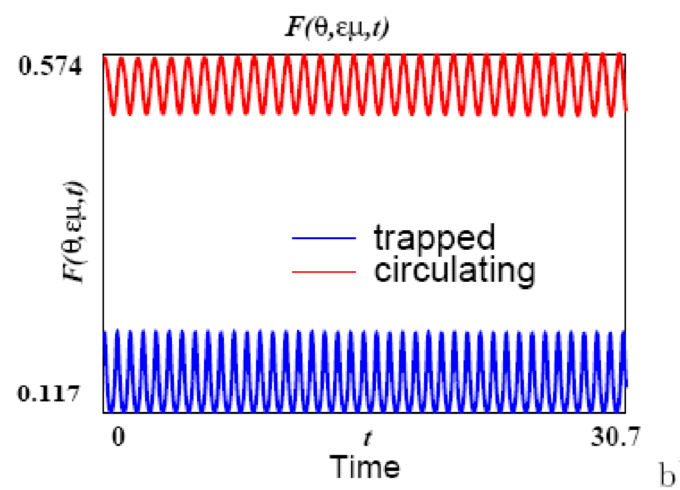


Spatial convection using 4th order upwinding and 5th order Weno scheme



t=0, t=1st cycle and t=14th cycles are overlapped, Nz=64



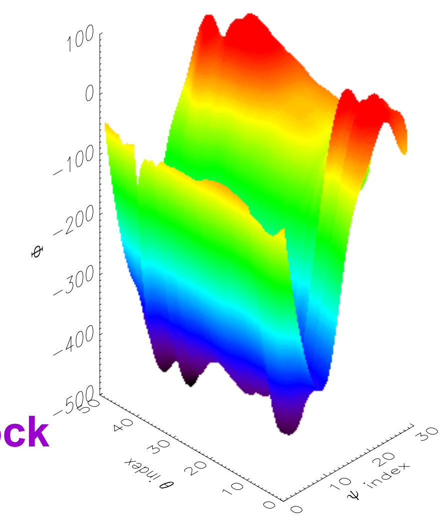


We have implemented a gyrokinetic Poisson equation field solver

$$\left(\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\right)\nabla_\perp^2\Phi + \left(\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\nabla_\perp \ln N_\alpha\right)\cdot\nabla_\perp\Phi + \nabla^2\Phi = -4\pi e\left(\sum_\alpha Z_\alpha N_\alpha - n_e\right) - \sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\frac{1}{N_\alpha Z_\alpha e}\nabla_\perp^2 p_{\perp\alpha}$$

$$N_e = N_{i0}\, e^{e\Phi/Te} / \langle e^{e\Phi/T_e} \rangle$$

- **Discretized in ψ-θ coordinates using standard finite differencing**
- **Uses *Hypre* library of parallel linear algebra solvers and preconditioners**
  - **Solvers:**
    - **Conjugate Gradient (CG)**
    - **Generalized Minimum Residual (GMRES)**
    - **Stabilized BiConjugate Gradient (BiCGSTAB)**
  - **Preconditioners**
    - **Diagonal scaling**
    - **Block Gauss-Seidel with PFMG or SMG in each block**
    - **BoomerAMG**



- **Currently implemented with both Boltzmann and kinetic electron model**

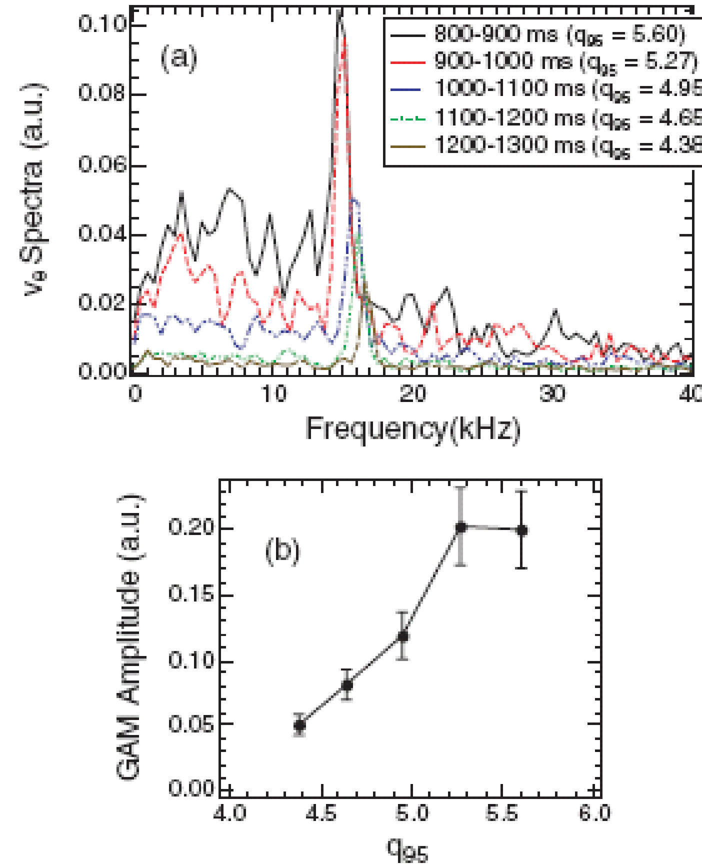
Tempest simulations of the radial electric field dynamics in neoclassical plasmas

- **Tempest simulations of collisionless damping of geodesic acoustic modes and zonal flow in uniform plasmas**

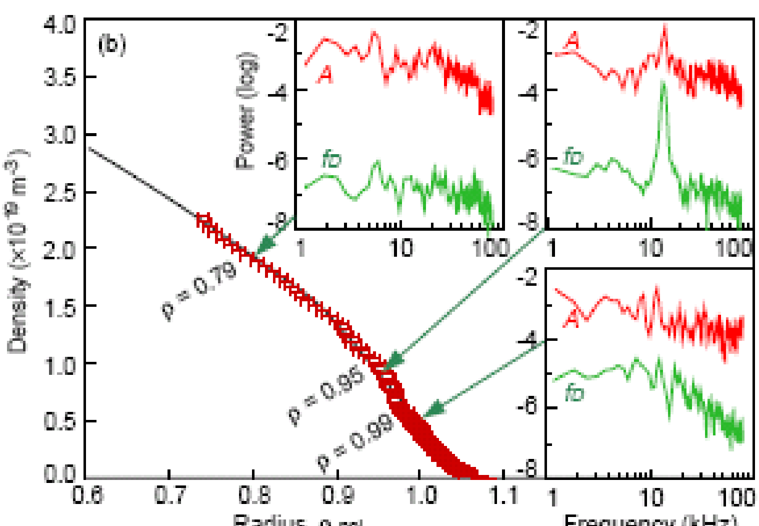
- **Tempest simulations of the radial electric field via Gyrokinetic Poisson Eq with steep gradients in the edge plasmas**

GAM is dominant in the edge plasmas

- **GAM and zonal flow has been clearly identified experimentally in tokamak and stellarator plasmas**
- **GAM is a coherent, radially localized poloidal flow oscillation that is dominant in the outer regions of the confined plasmas**
- **GAM and zonal flows are driven by the turbulence and act to regulate it via time-varying *E*×*B* flow shear de-correlation.**



DIII-D BES GAM expt.
Mckee, PPCF, 48, s123(2006)

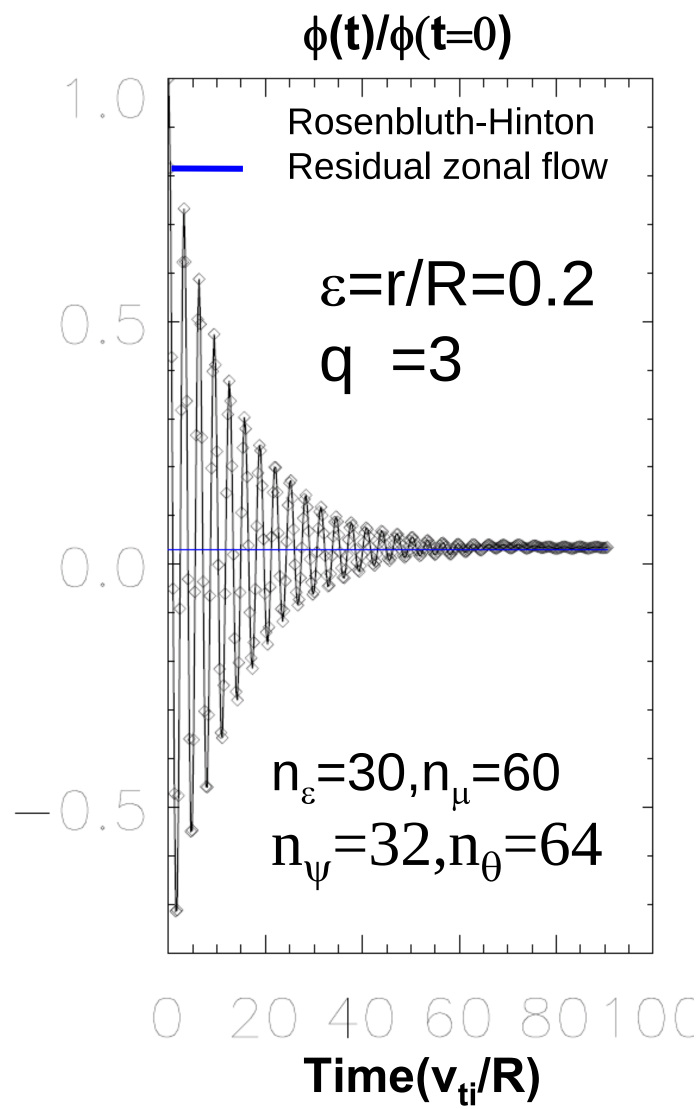


ASDEX-U
Conway, IAEA-CN-149/EX/2-1

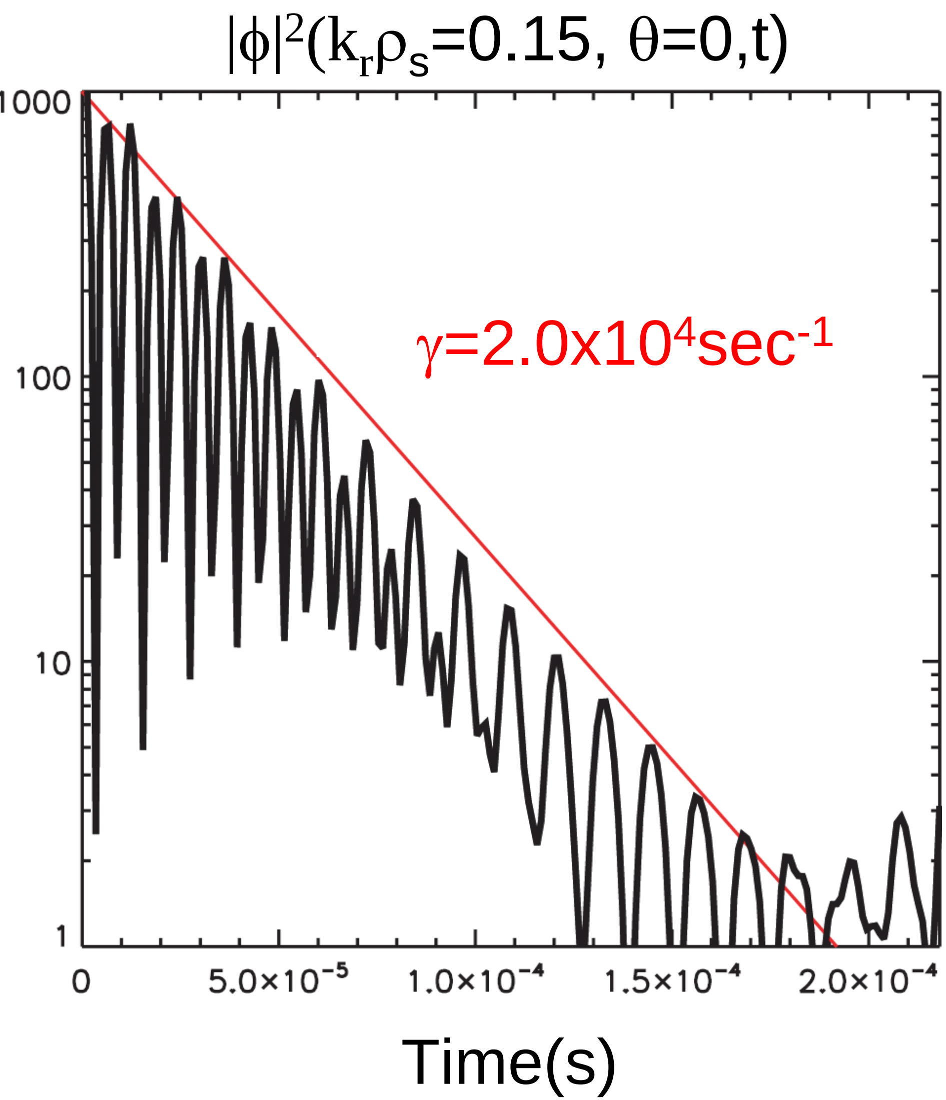
Tempest exhibits collisionless damping of GAM and zonal Flow

- **Tempest model**
  - **Drift kinetic ions with radial drift, streaming, and acceleration**
  - **Boltzmann electron**
  - **Gyrokinetic Poisson equation in limit small ρ_s/L_x**
  - **Periodic radial boundary conditions**
- **GAM provide opportunity to "verify" TEMPEST physics and to "extend" parameter regimes beyond analytical theory**
- **Simulation setup:**
  - **Homogeneous plasma with initial δn_i**

$$\delta n_i=\delta n_0 \sin(2\pi r/L_r)$$

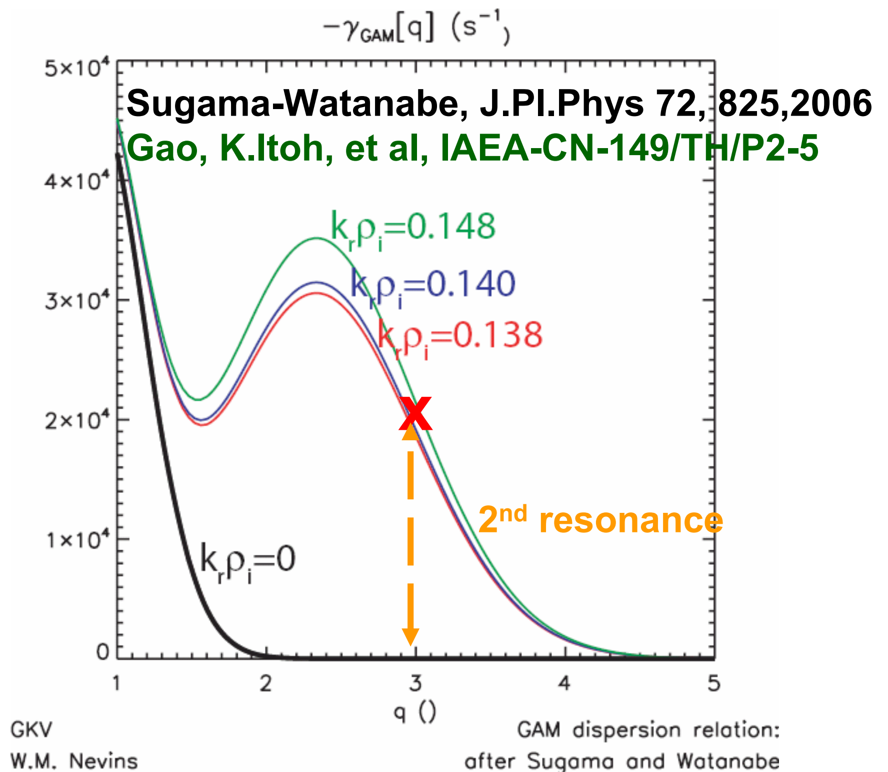


TEMPEST shows that damping of GAM follows theory with large banana orbits

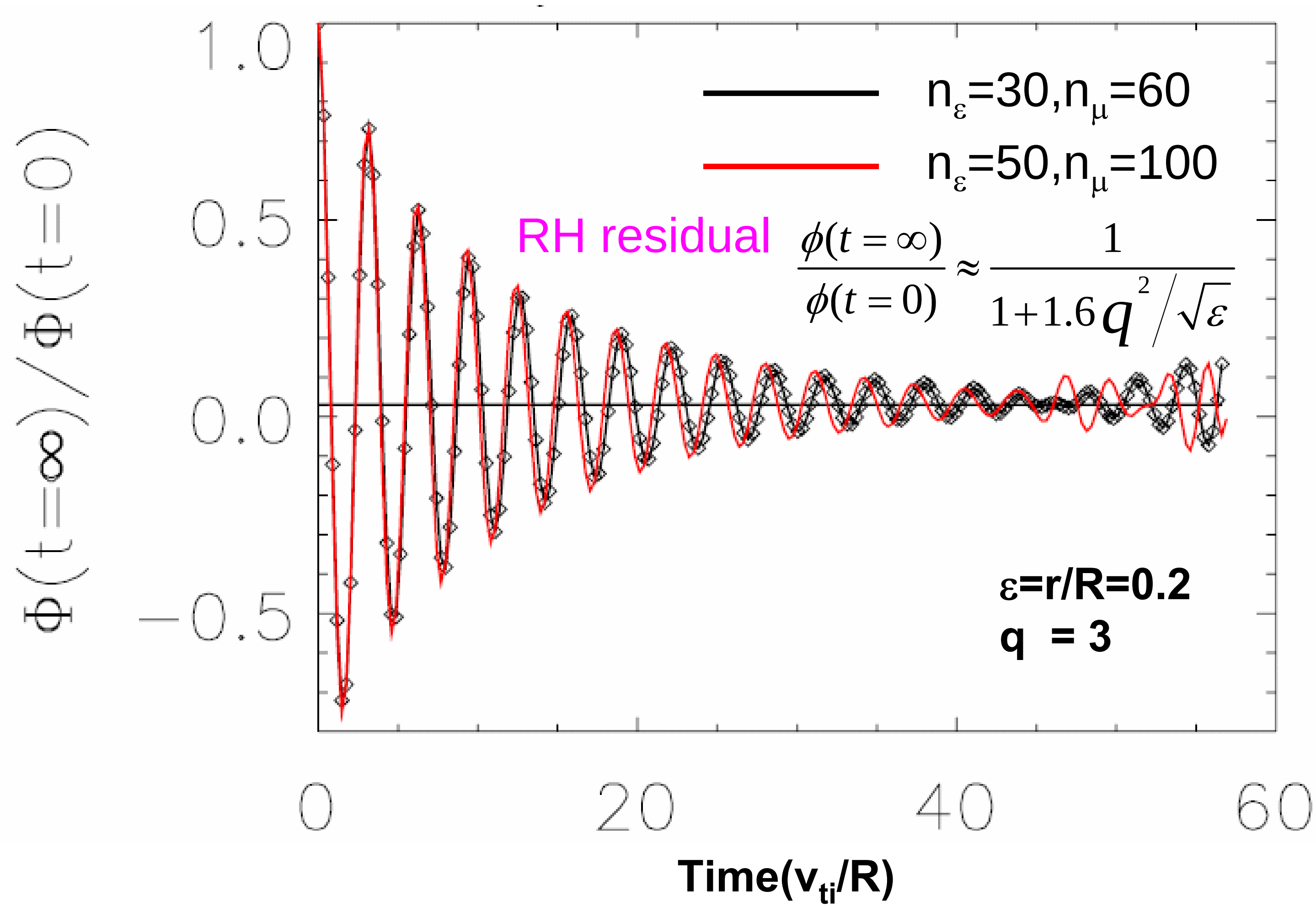


✓**Sugama & Watanabe show damping sensitive to $k\rho_i$ at large q (large banana orbit)**

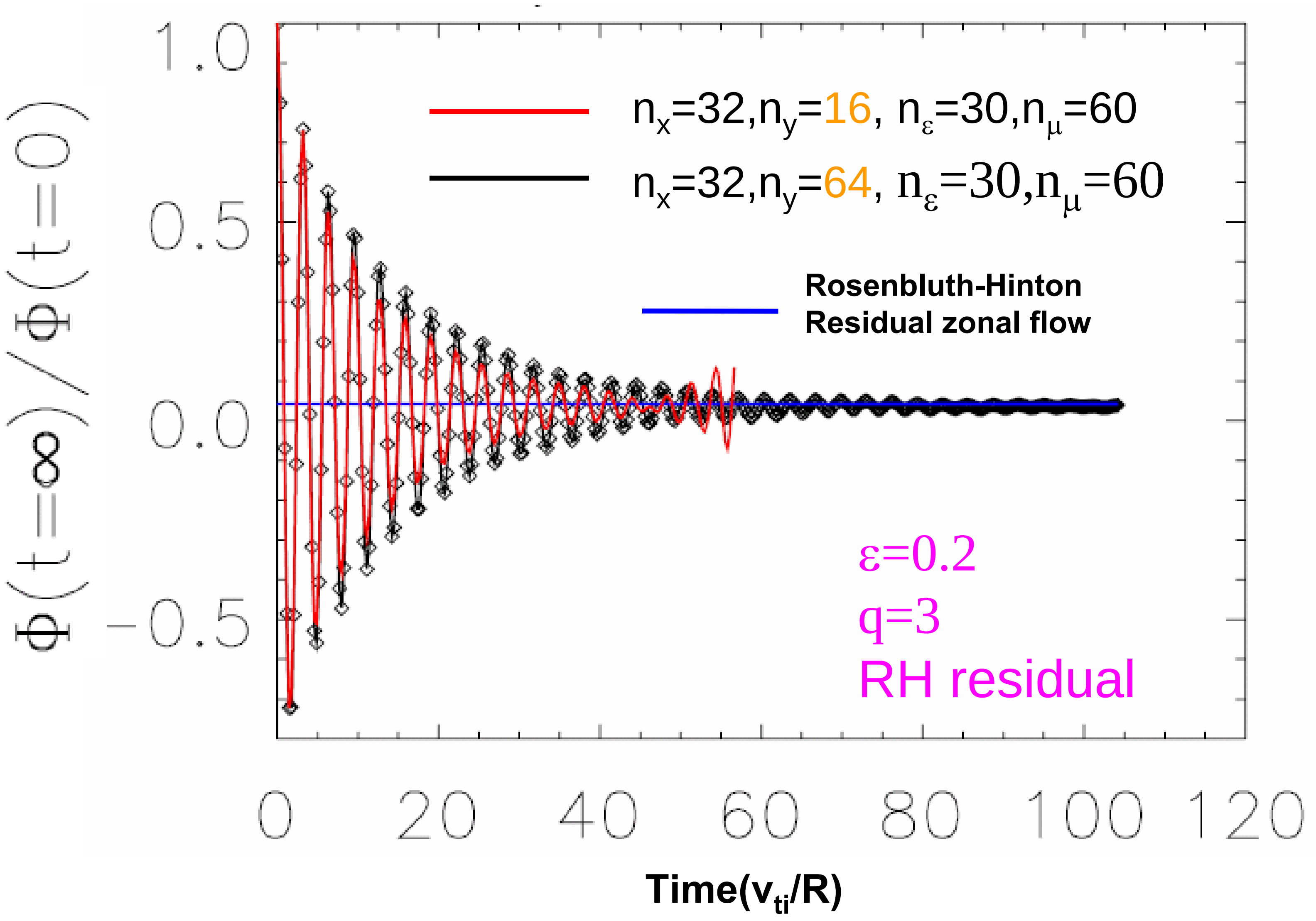
✓**Tempest shows reasonable agreement with theory, 17%.**



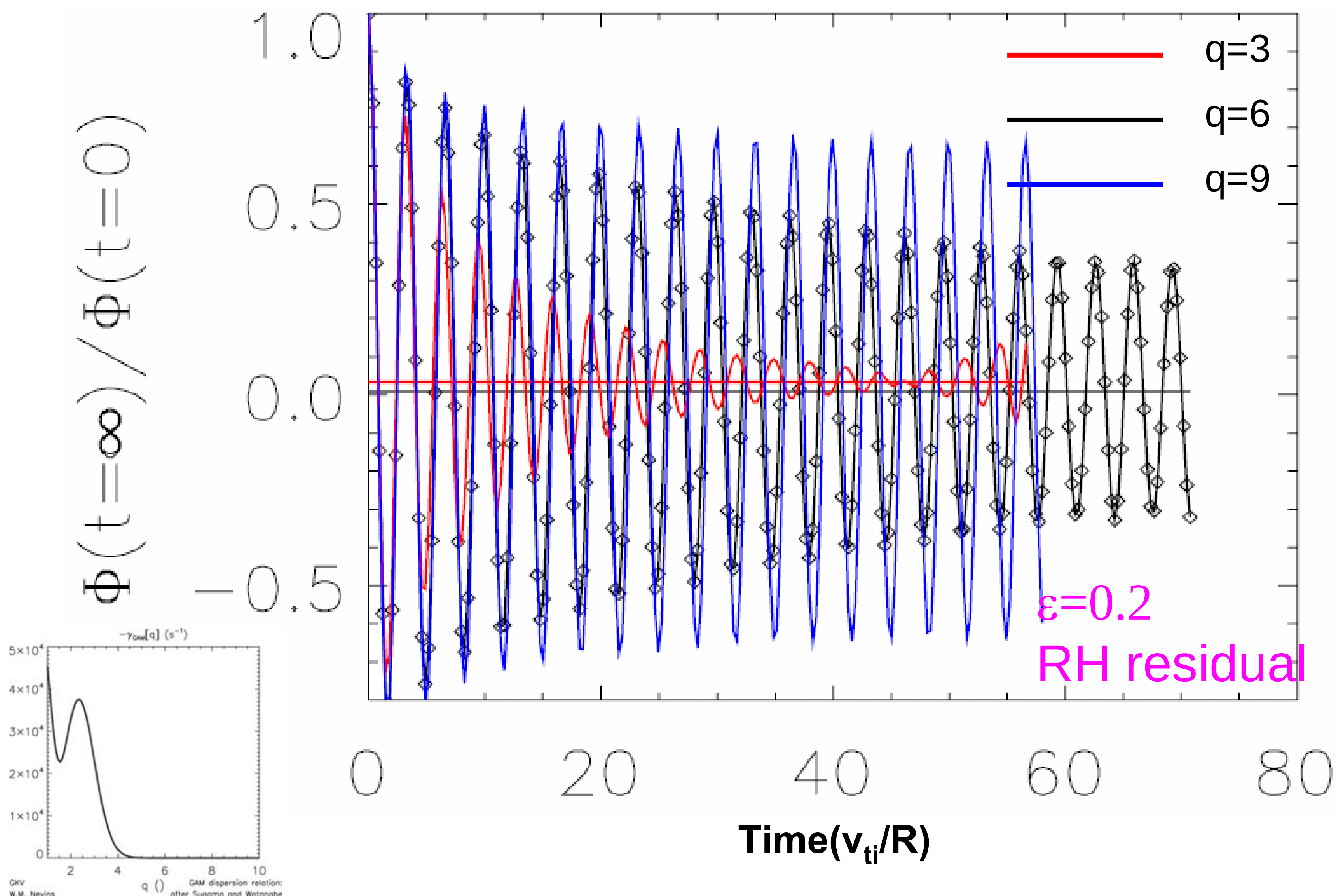
Frequency and damping rate in GAM simulation converge with velocity resolution



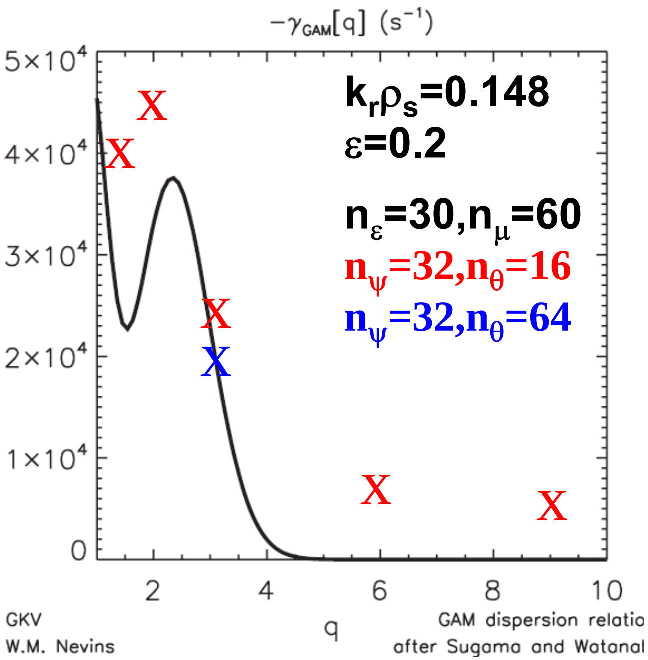
High poloidal resolution eliminates the recurrences

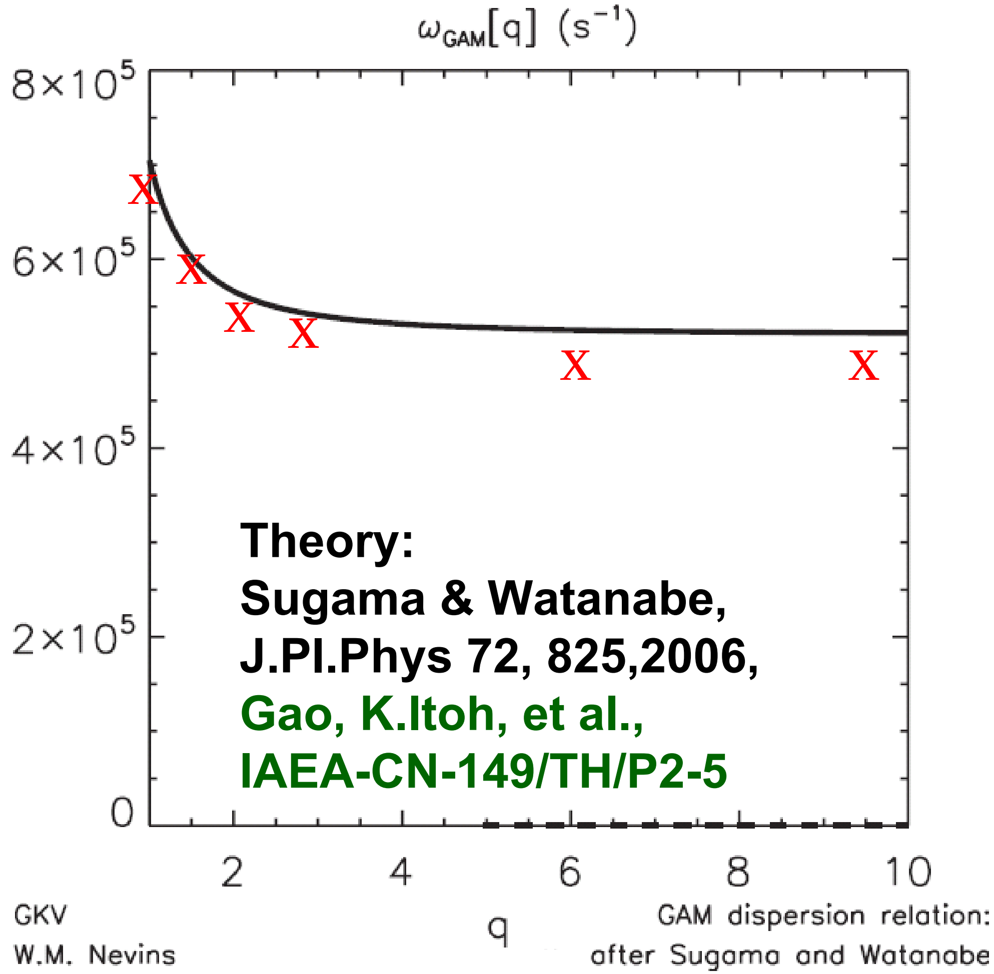


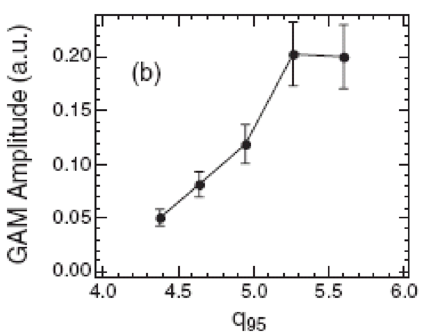
GAM simulation scan with q shows good agreement with RH residual



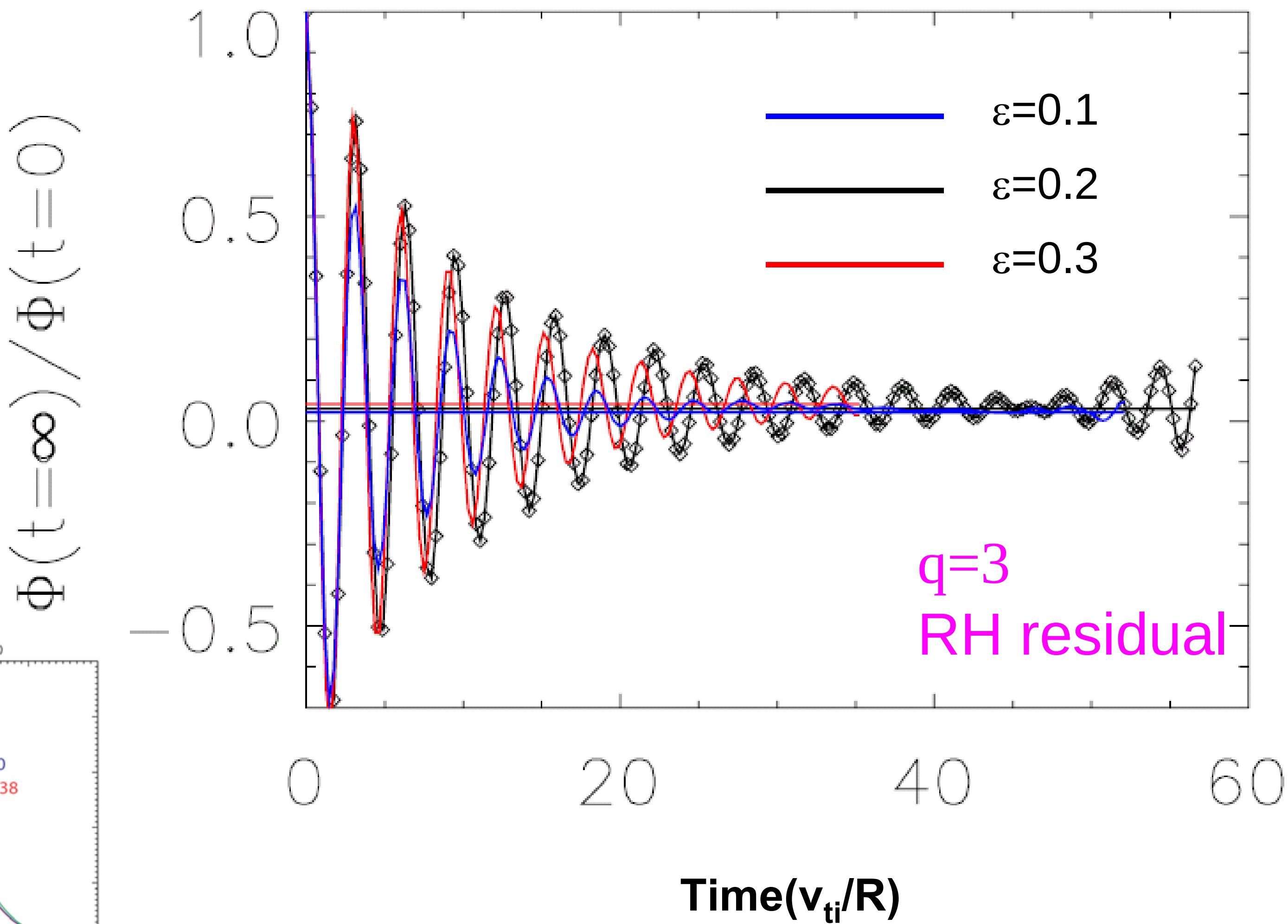
Frequency and damping rate vs q in GAM simulations in reasonable agreement with small ε theory

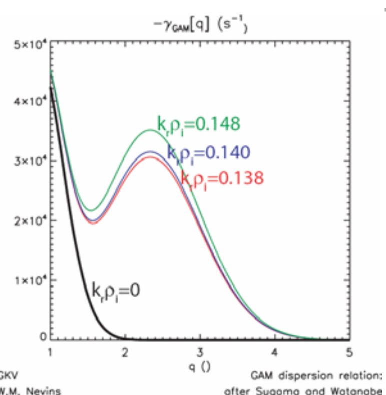




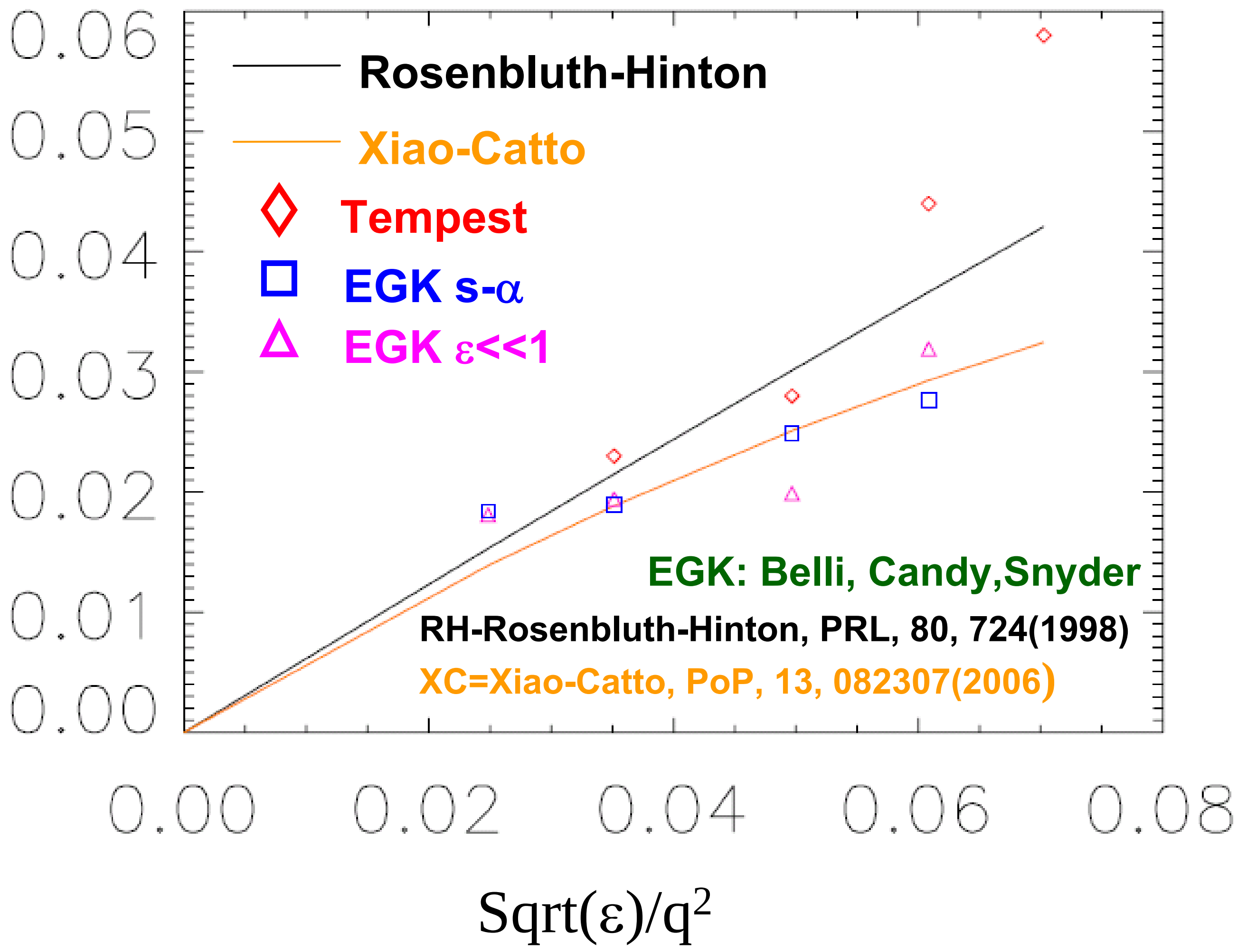


Residual vs ε in GAM simulations in good agreement with RH theory

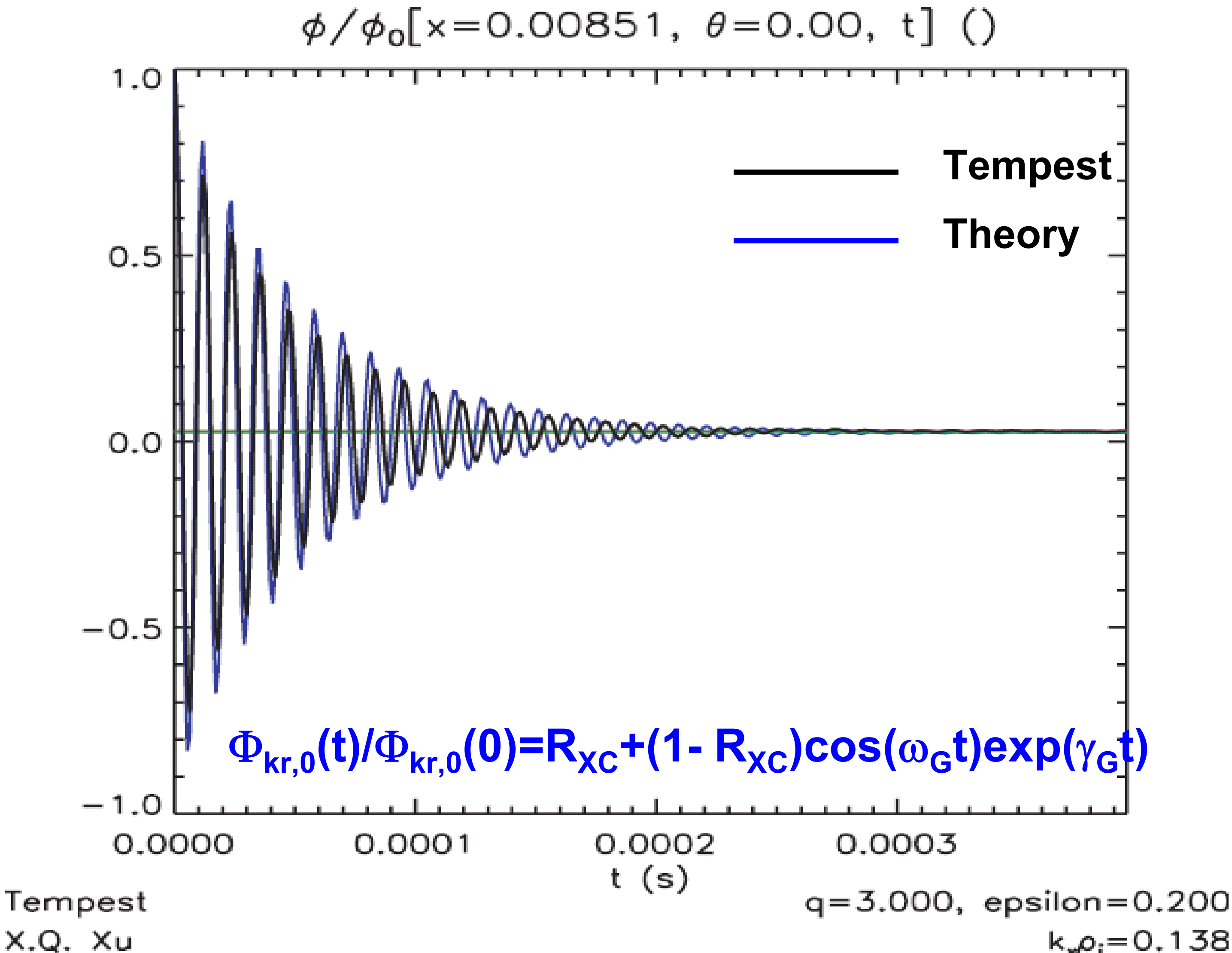




Residual vs ε in GAM simulations in reasonable agreement with theory



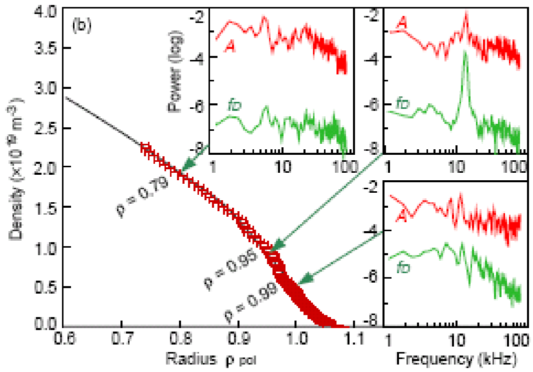
Tempest simulation shows good agreement theory



TEMPEST solves Gyrokinetic Poisson Eq in a steep gradient region

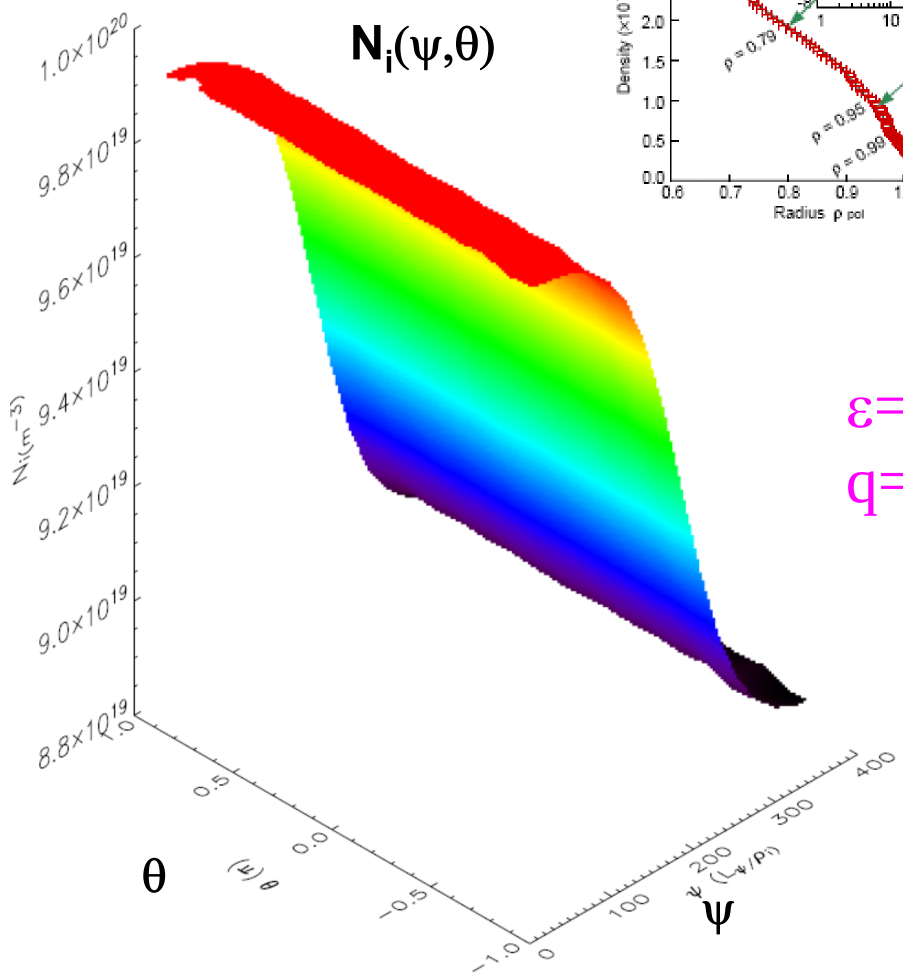
✓In circular geometry

✓Boltzmann & kinetic e^-



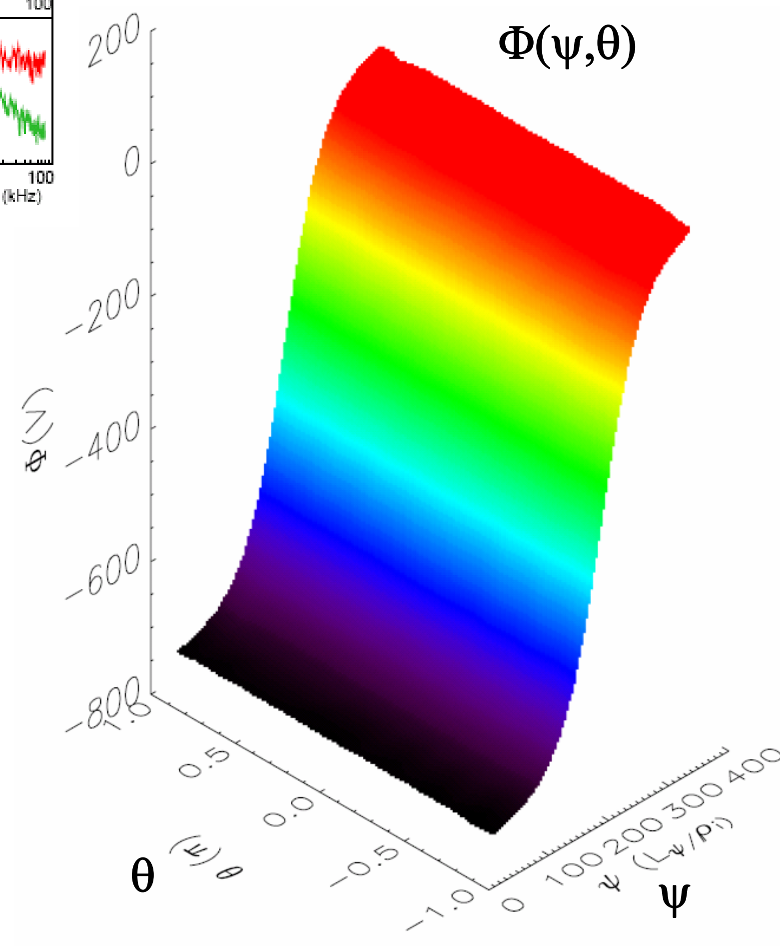
✓Krook collision

▪plateau regime

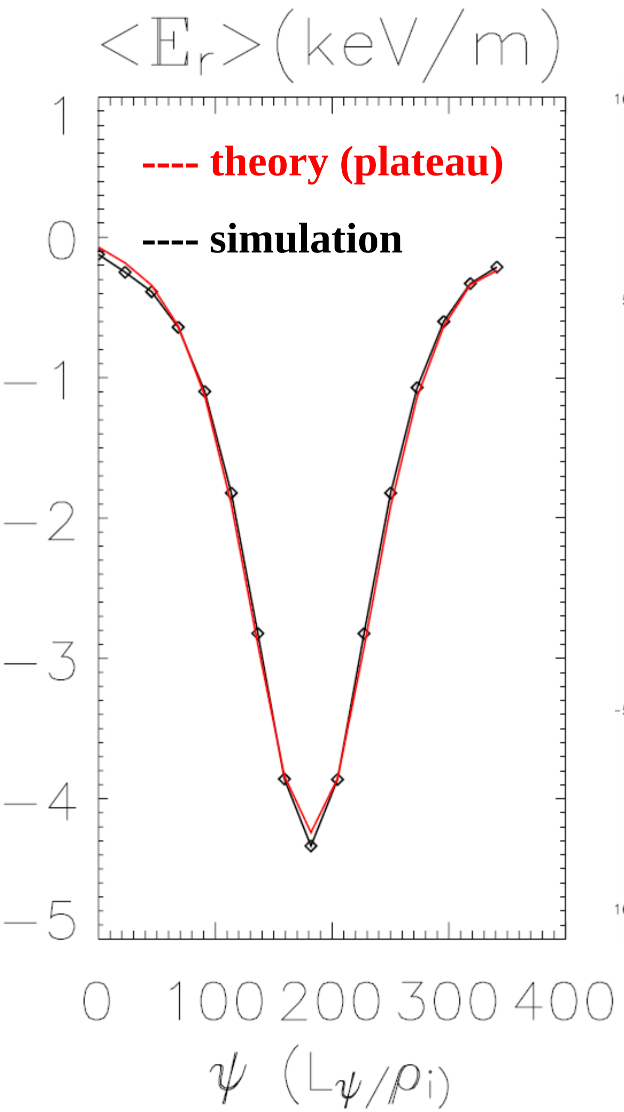


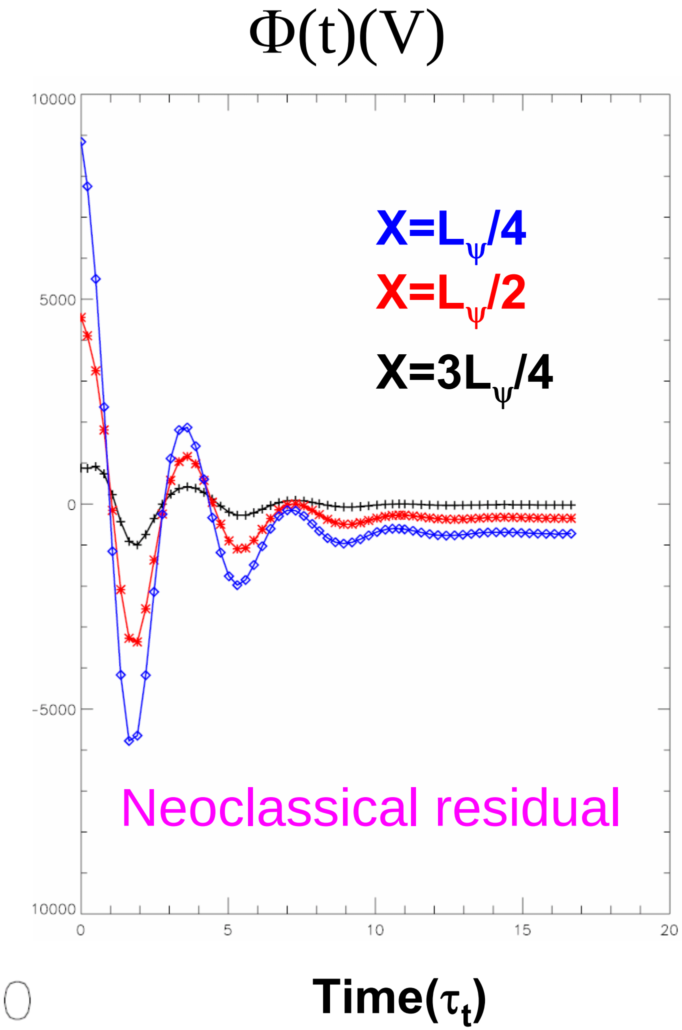
$\varepsilon=0.2$

$q=3$



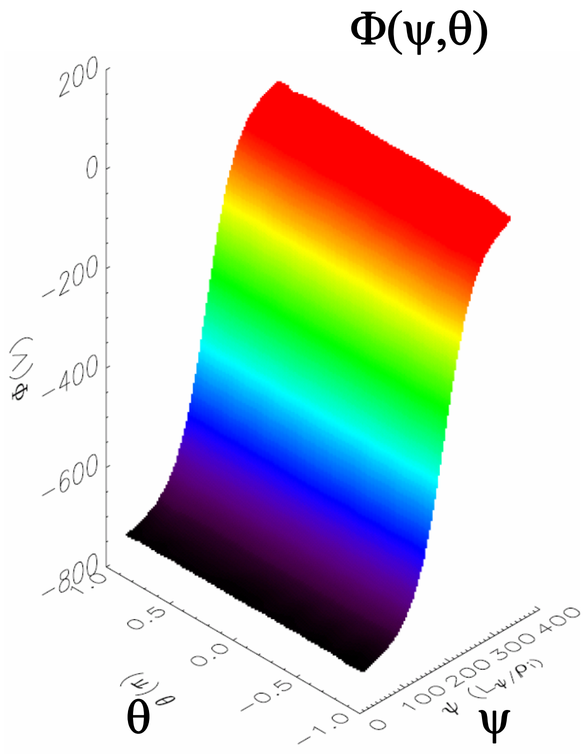
TEMPEST simulation show that initial GAM followed by self-consistent neoclassical Er



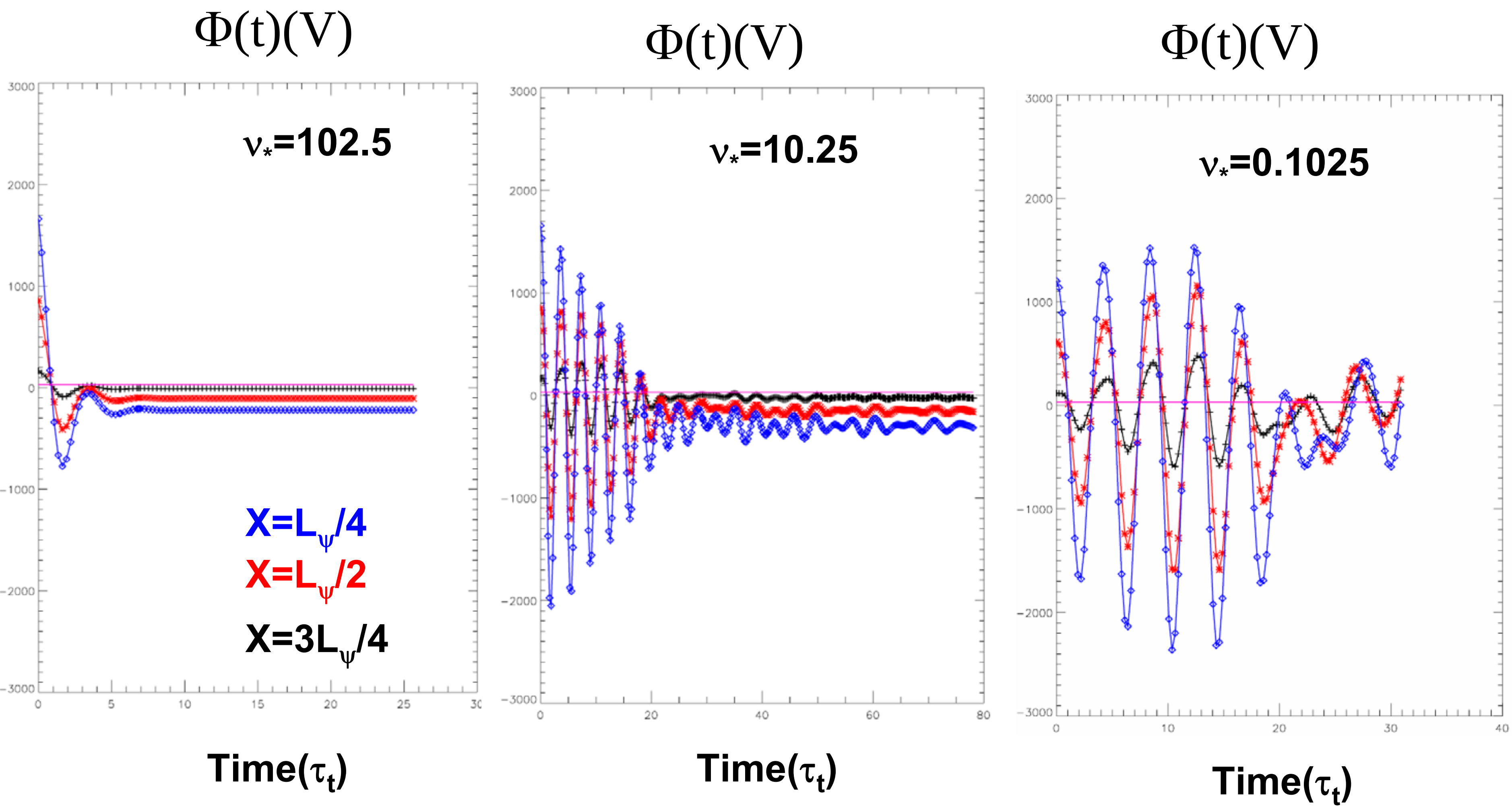


✓Er is generated due to neoclassical polarization

✓True steady state



TEMPEST shows GAM and collisional relaxation in edge plasma with steep gradients



Summary

- TEMPEST is a fully nonlinear (full-f) five dimensional (3d2v) gyrokinetic continuum edge-plasma code.
- The four-dimensional (2d2v) version of the code correctly produces frequency, collisionless damping of geodesic acoustic modes and zonal flow (Rosenbluth-Hinton residual) with Boltzmann electrons using a full-f code in small ε limit
- The electric field via Gyrokinetic Poisson Eq is also found to agree with the standard neoclassical expression for steep density and ion temperature gradients in the plateau regime with Boltzmann and kinetic electrons
- The preliminary encouraging results demonstrates the emerging capability of the TEMPEST code. The further improvement and development of TEMPEST will yield a valuable predictive model for the edge pedestal.