A velocity-dependent anomalous radial transport model for (2-D, 2-V) kinetic transport codes

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as part of

Edge Simulation Laboratory

Overview

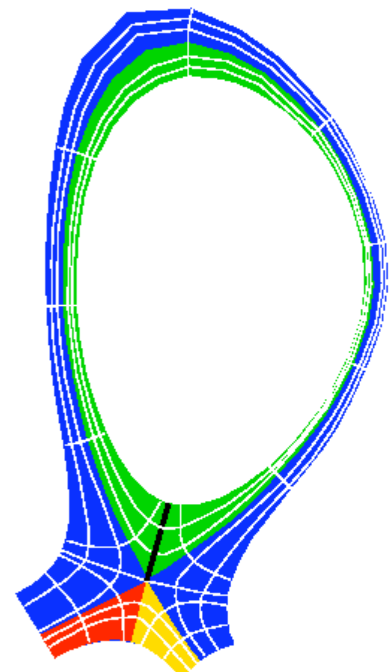
- TEMPEST is a 5-D kinetic code to simulate edge plasmas; runnable as a 4-D kinetic transport code.

- In order to perform quick-running studies of combined neoclassical and anomalous transport, we have added the following to TEMPEST:
  - an upgraded Krook collision model that simultaneously conserves density and energy
  - a model for anomalous transport

- This approach provides a possible starting point for a future self-consistent turbulence and transport model, where the model transport coefficients would be extracted from a simultaneously running 5D simulation.

Gyrokinetic equation has been implemented in the continuum TEMPEST for the edge

$$
\begin{aligned}
\frac{\partial F_\alpha}{\partial t} &+ \bar{\mathbf{v}}_{\rm d}\cdot\nabla_\perp F_\alpha + (\bar{v}_{\parallel\alpha} + v_{Banos})\nabla_\parallel \partial F_\alpha \\
&+ \left[q\frac{\partial\langle\Phi_0\rangle}{\partial t} + \bar{\mu}\frac{\partial B}{\partial t} - \frac{qB}{B^*}\bar{v}_\parallel \nabla_\parallel\langle\delta\phi\rangle - q\mathbf{v}_{\rm d}^0\cdot\bar{\nabla}\langle\delta\phi\rangle\right]\frac{\partial F_\alpha}{\partial E_0} \\
&= C(F_\alpha, F_\alpha),
\end{aligned}
$$



- GK F-equation discretized with high order (4th); Fokker-Planck collisions
- Full-f and δf options available
- Circular & divertor geom.; 2D equilibrium potential
- Runnable as
  - 4-D for transport with F(Ψ,θ,ε,μ), or
  - 5-D for turbulence with F(Ψ,θ,ϕ ,ε,μ) (poster 122 in this session)
- Extensions planned:
  - sources/sinks
  - model transport coefficients for initial anomalous transp.
  - generalized GK equations (see Qin)
  - optional fluid equations in same framework
  - *field-aligned coordinates for evolving B

This poster

Formulation, implementation, and testing of an anomalous radial diffusion operator

Our goal is to add a model for turbulent transport that can be combined with neoclassical ion transport

- **need a diagonal transport matrix for comparison with fluid models**

$$h_\psi \Gamma_n = -D_n \frac{\partial n}{\partial \psi} \qquad h_\psi Q = -\frac{5}{2} D_n T \frac{\partial n}{\partial \psi} - \chi n \frac{\partial T}{\partial \psi}$$

Model the turbulent transport as a combination of advection and diffusion, as is conventionally done in fluids

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial s} + v_d \cdot \nabla f + E_\parallel \frac{\partial f}{\partial v_\parallel} + \frac{1}{V} \left. \frac{\partial \left(V \, \Gamma_a / h_\psi \right)}{\partial \psi} \right|_{\theta, v} = S$$

$$\Gamma_a = U_a f - \frac{D}{h_\psi} \left. \frac{\partial f}{\partial \psi} \right|_{\theta, v}$$

– U_a, D depend on position (ψ) and velocity (v)

- specifying different velocity dependence allows separate control of "D_n" and "χ" (and mom. coeff.)
- advection U_a allows $D(v)$ to be positive for all velocities
- provides flexibility, speed (compared to coupling turbulence), and comparison to fluid models

Modeling the transport coefficients

Define convective coefficient $U_a(\psi) = \frac{1}{h_\psi}\left(\alpha \frac{\partial \ln n}{\partial \psi} + \beta \frac{\partial \ln T}{\partial \psi}\right)$

Define diffusive coefficient $D(\psi, \hat{\mathrm{v}}) = D_0 + D_2\left(\hat{\mathrm{v}}^2 - \tfrac{1}{2}\right) + D_4\left(\hat{\mathrm{v}}^4 - 3\hat{\mathrm{v}}^2 + \tfrac{3}{4}\right)$

where $\left(\hat{\mathrm{v}} = \mathrm{v}/\mathrm{v}_{th}\right)$

Particle flux $h_\psi \Gamma_n = -(D_0 + D_2 - \alpha)\frac{\partial n}{\partial \psi} - \frac{3}{2}(D_2 + 2D_4 - \frac{2}{3}\beta) n \frac{\partial \ln T}{\partial \psi}$

&

Heat flux $h_\psi Q = -\frac{3}{2}\left\{(D_0 + 2D_2 + 2D_4 - \alpha) T \frac{\partial n}{\partial \psi} + \left(D_0 + \frac{9}{2} D_2 + 12 D_4 - \beta\right) n \frac{\partial T}{\partial \psi}\right\}$

- Particle flux not directly dependent on the local temperature gradients
- Particle flux leads to a corresponding heat flux (specific heat)

Modeling the transport coefficients

Diagonal form of the transport matrix for

$$\beta = D_n \qquad D_4 = \frac{1}{5}\left(\frac{2}{3}\chi - \frac{4}{3}\beta - D_n - \alpha\right)$$

$$D_2 = 2\left(\frac{1}{3}\beta - D_4\right) \qquad D_0 = D_n + \alpha - D_2$$

α can be chosen to ensure positivity of the diffusion coefficient

– For the diffusion coefficient with a simple quadratic dependence over speed (v), we choose

$$\alpha = \frac{2}{3}\left(\chi - \frac{7}{2}D_n\right)$$

$$D_2 = \frac{2}{3}D_n \qquad D_0 = \frac{1}{3}D_n + \alpha$$

– Diffusion coefficient is non-negative over the velocity domain if

$$\alpha \geq 0 \quad \rightarrow \quad \chi \geq \frac{7}{2}D_n$$

$$\boxed{h_\psi \Gamma_n = -D_n \frac{\partial n}{\partial \psi} \qquad h_\psi Q = -\frac{5}{2}D_n T \frac{\partial n}{\partial \psi} - \chi n \frac{\partial T}{\partial \psi}}$$

Anomalous radial transport: Implementation

- Velocity coordinates are *(ε_o, μ)*
  - derivative at constant **v** not the same as derivative at constant (ε_o, μ)

$$\left.\frac{\partial f}{\partial \psi}\right|_{\theta,\mathrm{v}} = \left.\frac{\partial f}{\partial \psi}\right|_{\theta,\varepsilon,\mu} + q\frac{\partial \Phi}{\partial \psi}\frac{\partial f}{\partial \varepsilon} - \frac{\mu}{B}\frac{\partial B}{\partial \psi}\frac{\partial f}{\partial \mu}$$

- Kinetic equation is now second-order differential equation in space
  - boundary conditions enforced for incoming & outgoing particles at radial boundaries
  - diffusion evaluated using a 2nd order Central Differencing Scheme
- Contribution to radial transport computed by computing moments of the flux (Γ_a) over the velocity (ε_o, μ) space

Krook Collision Model: Implementation

Computation of radial transport of particles and energy requires a number- and energy-conserving collision model. In order to have one that is reasonably fast, we implement an upgrade of TEMPEST's Krook model that simultaneously conserves energy and particle density

$$\left.\frac{\partial f}{\partial t}\right|_c = -\tilde{o}_k\left[f(\mathrm{v}) - f_{Max}(\mathrm{v})\right]$$

$$\int \mathrm{d}\mathbf{v}\, f(\mathbf{v}) = \int \mathrm{d}\mathbf{v}\, f_M(\mathbf{v})$$

$$\int \mathrm{d}\mathbf{v}\, \mathrm{v}^2 f(\mathbf{v}) = \int \mathrm{d}\mathbf{v}\, \mathrm{v}^2 f_M(\mathbf{v})$$

$$\left.\frac{\partial n}{\partial t}\right|_c = -\tilde{o}_c \int \mathrm{d}\mathbf{v}\left[f(\mathbf{v}) - f_{Max}(\mathbf{v})\right] = 0$$

$$\left.\frac{\partial}{\partial t}(nT)\right|_c = -\tilde{o}_c \int \mathrm{d}\mathbf{v}\, \mathrm{v}^2\left[f(\mathbf{v}) - f_{Max}(\mathbf{v})\right] = 0$$

- f_{Max} is a Maxwellian corresponding to the local density and temperature
- model conserves energy & particle density

Moments of the distribution function are needed to update the Maxwellian f_{Max}

- Numerical error in the moment computation affects f_{Max} and hence, the conservative character of the collision term
- Moments are computed twice at each time step to account for this error

$$(n, T) \rightarrow f(\mathrm{v}) \xrightarrow{\text{num}} (n_0, T_0) \rightarrow f_M(\mathrm{v}) \xrightarrow{\text{num}} (n_1, T_1)$$

$$\int_{\text{num}} \mathrm{d}\mathbf{v}\, f(\mathrm{v}) \neq \int_{\text{num}} \mathrm{d}\mathbf{v}\, f_M(\mathrm{v})$$

$\rightarrow$

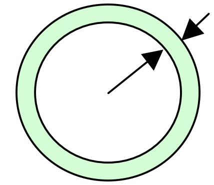
We do not know *n*, *T*.
Assume $n/n_0 = n_0/n_1$

$$\left(n_0^2/n_1, T_0^2/T_1\right) \rightarrow f_M(\mathrm{v})$$

- Error is reduced, but not eliminated
- Computing moments twice each time-step is computationally expensive

Anomalous Diffusion + Krook Collision: Test case

- Diffusion and collision model implementations were tested on an annular geometry



  - small aspect ratio → cylindrical geometry
  - domain is periodic in the poloidal direction
  - Krook collision term is computed to check conservation/cost
  - initially radial/poloidal drift switched off → diffusion in an annulus

$$\frac{\partial f}{\partial t} = -\frac{1}{V}\frac{\partial}{\partial \psi}\left[\frac{V}{h_\psi}\frac{D(\psi,\mathrm{v})}{h_\psi}\frac{\partial f}{\partial \psi}\right] - \tilde{o}_k\left[f(\mathrm{v}) - f_{Max}(\mathrm{v})\right]$$

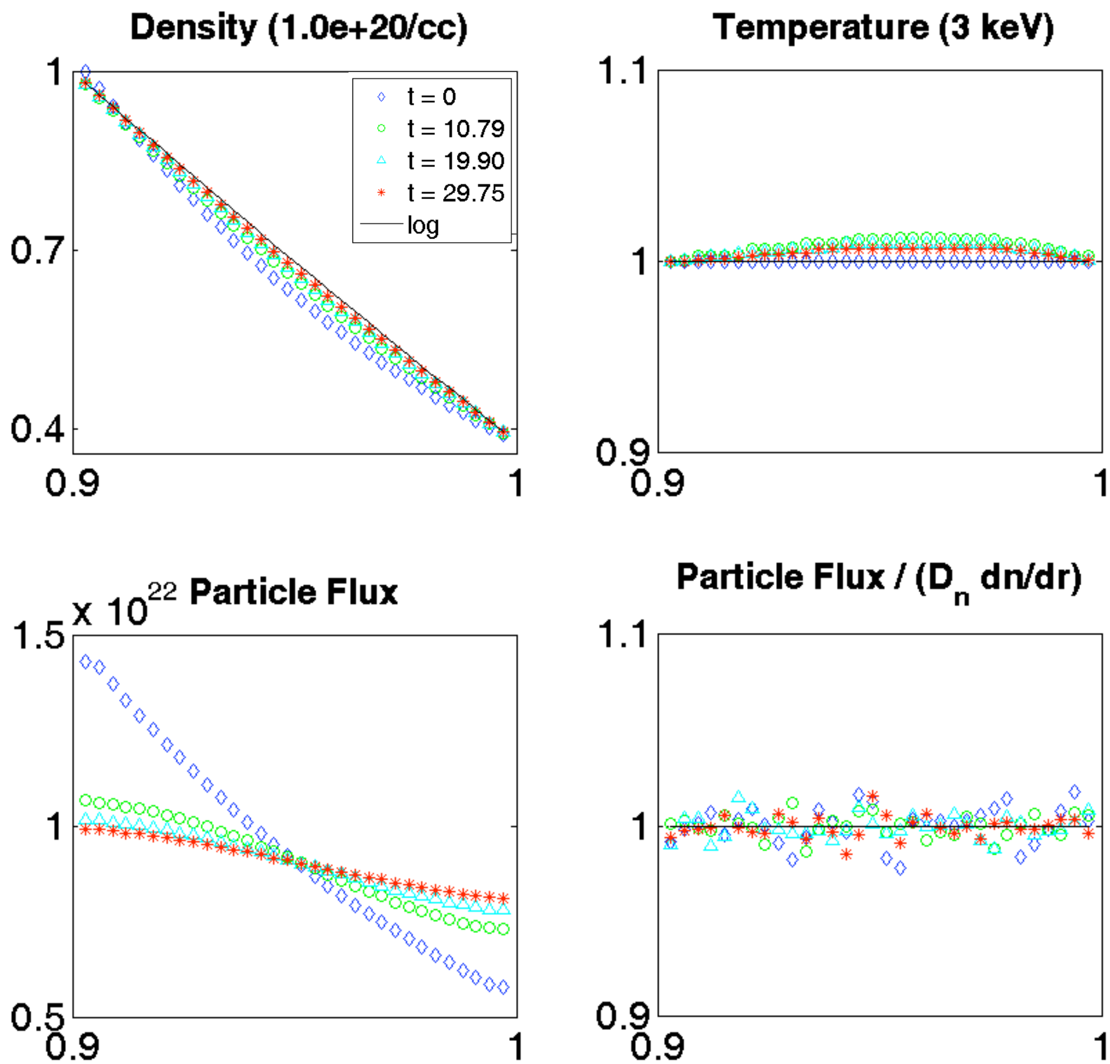
$$V = 2\pi R h_\psi h_\theta$$

  - **Case I**: uniform temperature with a radial density gradient
  - **Case II**: radial temperature gradient with uniform density

- Simulation parameters:
  - domain width is 0.1*minor radius
  - spatial grid: 32(radial) by 8(poloidal)
  - Max kinetic energy (velocity space extent): 16*T
  - Velocity space grid: 65 (ε_o) by 50 (μ)

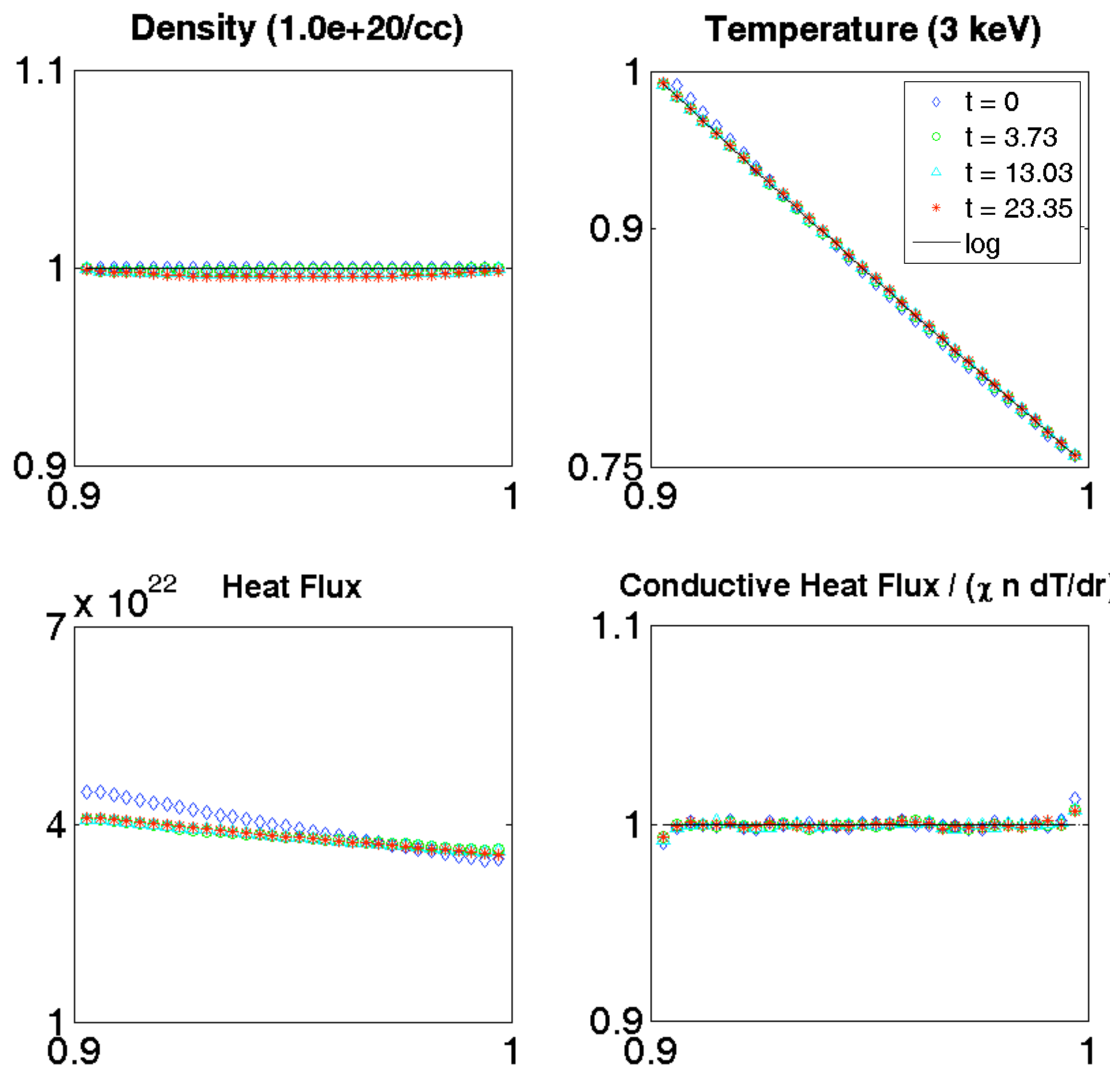
I. Radial density gradient, uniform temperature

- Diffusion model is defined by a diffusivity of 10 m^2/s and conductivity of 35 m^2/s
- Density is initialized to an exponential profile
- Density approaches a logarithmic variation, as is expected at steady-state
- Temperature begins to relax back to its initial state

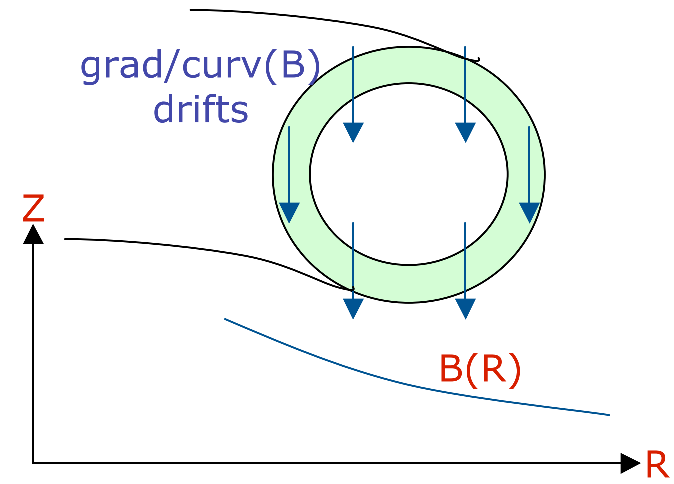


II. Radial temperature gradient, uniform density

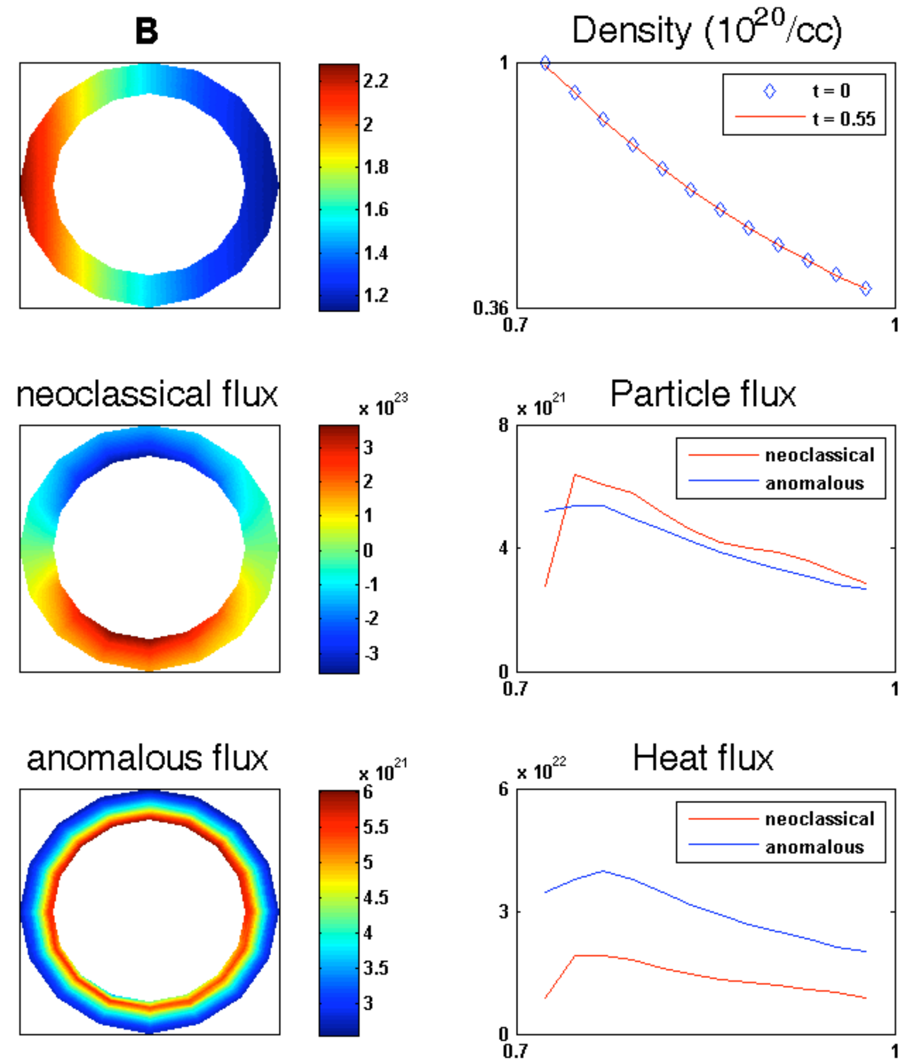
- Diffusion model is defined by a diffusivity of 10 m^2/s and conductivity of 35 m^2/s
- Temperature reaches a steady-state early, and remains unchanged
- Density yet to relax to initial state, unlike the temperature in earlier case



Neoclassical & Anomalous radial transport



- In this test simulation, we combine the anomalous transport model with radial drifts and the Krook collision model. There is no radial electric field in this case, and hence the drifts are due to grad(B) and curv(B).
- We can see that, though the flux due to the drifts is two orders of magnitude greater locally, the poloidally averaged radial transport due to the drifts is of the same order as anomalous transport.



Summary

- An anomalous radial transport model in the form of a combination of convective and diffusive transport has been added to TEMPEST
  - the model is in the form of a diagonal transport matrix; transport coefficients can be assigned so as to be equivalent in the highly collisional limit to fluid models; allows comparison with fluid models.
  - in the future the transport coefficients could be extracted from a simultaneously running 5D simulation
- The Krook collision model in TEMPEST has been upgraded to make it more suitable for simultaneous neoclassical and anomalous radial transport calculations
  - particle/energy conservation in the Krook collision term requires moment computation more than once to improve accuracy
  - multiple moment computation is computationally expensive
  - need a less expensive model (one moment computation per time-step)

$$f_M(\mathrm{v}) = a_1 f_{M_1}(\mathrm{v}) + a_2 f_{M_2}(\mathrm{v})$$

Computed only at t=0

$$f_{M_i}(\mathrm{v}) \xrightarrow{\text{num}} (n_i, T_i)$$

at current time-step

$$f(\mathrm{v}) \xrightarrow{\text{num}} (n, T)$$

$$\longrightarrow$$

$$a_1 = \frac{n}{n_1}\left(\frac{T_2 - T}{T_2 - T_1}\right); \qquad a_2 = \frac{n}{n_2}\left(\frac{T - T_1}{T_2 - T_1}\right)$$

$$f_M(\mathrm{v}) \xrightarrow{\text{num}} (n, T) \xleftarrow{\mathit{num}} f(\mathrm{v})$$