Status of Tempest Development for 5D Edge Gyrokinetic Simulations

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Fully nonlinear ion gyrokinetic equation has been cast in E^0-μ coordinates

$$
\begin{aligned}
\frac{\partial F_\alpha}{\partial t} &+ \bar{\mathbf{v}}_{\mathrm{d}} \cdot \frac{\partial F_\alpha}{\partial \bar{\mathbf{x}}_\perp} + (\bar{v}_{\|\alpha} + v_{Banos})\mathbf{b} \cdot \frac{\partial F_\alpha}{\partial \bar{\mathbf{x}}} \\
&+ \left[q\frac{\partial \langle \Phi^0 \rangle}{\partial t} + \bar{\mu}\frac{\partial B}{\partial t} - \frac{B}{B^*}\bar{v}_\| q \frac{\partial \langle \delta\phi \rangle}{\partial s} - \mathbf{v}_{\mathrm{d}}^0 \cdot (q\bar{\nabla}\langle \delta\phi \rangle) \right] \frac{\partial F_\alpha}{\partial E_0} \\
&= C(F_\alpha, F_\alpha), \\
\bar{\mathbf{v}}_{\mathrm{d}} &= \frac{c\mathbf{b}}{qB_\|^*} \times \left(q\bar{\nabla}\langle \Phi \rangle + \bar{\mu}\bar{\nabla}B \right) + \bar{v}_\|^2 \frac{M_\alpha c}{qB_\|^*}(\bar{\nabla} \times \mathbf{b}). \\
\bar{\mathbf{v}}_{\mathrm{d}}^0 &= \frac{c\mathbf{b}}{qB_\|^*} \times \left(q\bar{\nabla}\langle \Phi^0 \rangle + \bar{\mu}\bar{\nabla}B \right) + \bar{v}_\|^2 \frac{M_\alpha c}{qB_\|^*}(\bar{\nabla} \times \mathbf{b}). \\
\bar{v}_\| &= \pm\sqrt{\frac{2}{M_\alpha}(E_0 - \bar{\mu}B - q\langle \Phi^0 \rangle)}, \\
v_{Banos} &= \frac{\mu c}{q}(\mathbf{b} \cdot \bar{\nabla} \times \mathbf{b}), \\
B_{\|\alpha}^* &\equiv B\left[1 + \frac{\mathbf{b}}{\Omega_{c\alpha}} \cdot (v_\| \bar{\nabla} \times \mathbf{b}) \right], \Omega_{c\alpha} = \frac{qB}{M_\alpha c}, \mu = \frac{\mathrm{M}_\alpha \mathrm{v}_\perp^2}{2\mathrm{B}}, \\
\langle \delta\phi \rangle &= \langle \Phi \rangle - \langle \Phi^0 \rangle.
\end{aligned}
$$

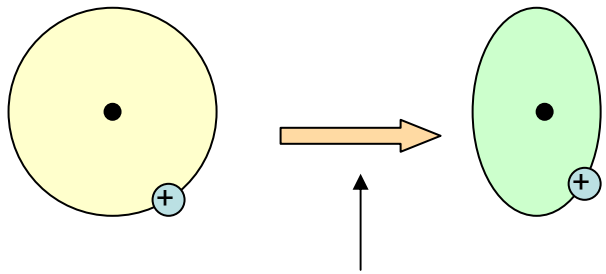
- ✓**The field is split into two parts: Φ^0 and $\delta\phi$;**
- ✓**$E^0 = mv^2/2 + q\Phi_0$, a constant of motion if $\delta\phi \sim 0$ and a coordinate aligned with flow;**
- ✓**E^0xB flow terms and the slow variation Φ^0 from Qin's formulation will be added.**

Fully nonlinear gyrokinetic Poisson equation--- in collaboration with Hong Qin

$$\nabla^2\phi(\mathbf{x}) = -4\pi\sum_s q_s\left[N + N_{\phi_0} + N_{\phi_1}\right],$$

$$N(\mathbf{x}) \equiv \int 2\pi w\,dwdu\; I_0\left(\rho\nabla_\perp\right)F\left(\mathbf{x},w,u\right),$$

$$N_{\phi_0}(\mathbf{x}) \equiv \frac{1}{B_0^2}\left(\mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2\right)\nabla\left[n(\mathbf{x})\left(\mathbf{D} + V_\parallel(\mathbf{x})\mathbf{b}\right)\cdot\nabla\mathbf{D}\right],$$



Orbit squeezing by large Er shear

$$N_{\phi_1}(\mathbf{x}) \equiv -\phi_1(\mathbf{x})\sum_{i=1}^{\infty}\frac{2i}{\left(i!\right)^2}\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^i M_{2i-2}(\mathbf{x})$$
$$+\sum_{i,j=0}^{\infty}\frac{2\left(i+j\right)}{\left(i!j!\right)^2}\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^i\left[M_{2(i+j)-2}(\mathbf{x})\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^j\phi_1(\mathbf{x})\right],$$

Full FLR effect

$$n(\mathbf{x}) \equiv \int 2\pi w\,dwdu\; F\left(\mathbf{x},w,u\right),$$

$$V_\parallel(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\int 2\pi w\,dwdu\; F\left(\mathbf{x},w,u\right),$$

$$M_i(\mathbf{x}) \equiv \int 2\pi w\,dwdu\; w^i F\left(\mathbf{x},w,u\right).$$

$$I_0\left(\rho\nabla_\perp\right) \equiv \sum_{i=0}^{\infty}\frac{1}{\left(i!\right)^2}\left(\frac{\nabla_\perp^2 w^2}{4\Omega_0^2}\right)^i$$

$F\left(\mathbf{x},w,u\right)$ -- total gyrocenter distribution function.

Evaluated at the particle coordinates.

Qin, Cohen, Nevins, and Xu, Contrib. Plasma Phys. 46, 7-9, 477 (2006)

Fully nonlinear gyro-kinetic Poisson equation in the long wavelength limit

In the long wavelength limit $k_\perp \rho_\alpha \ll 1$, the self-consistent electromagnetic field are typically computed from the gyro-kinetic Poisson equation for the multiple species

$$\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\frac{1}{N_\alpha}(\nabla_\perp \cdot N_\alpha \nabla_\perp \Phi) + \nabla^2\Phi = -4\pi e\left[\sum_\alpha Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right] - \sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\frac{1}{N_\alpha Z_\alpha e}\nabla_\perp^2 p_{\perp\alpha}.$$

where the gyrocenter center density N_α and perpendicular ion pressure $p_{\perp\alpha}$ are defined by

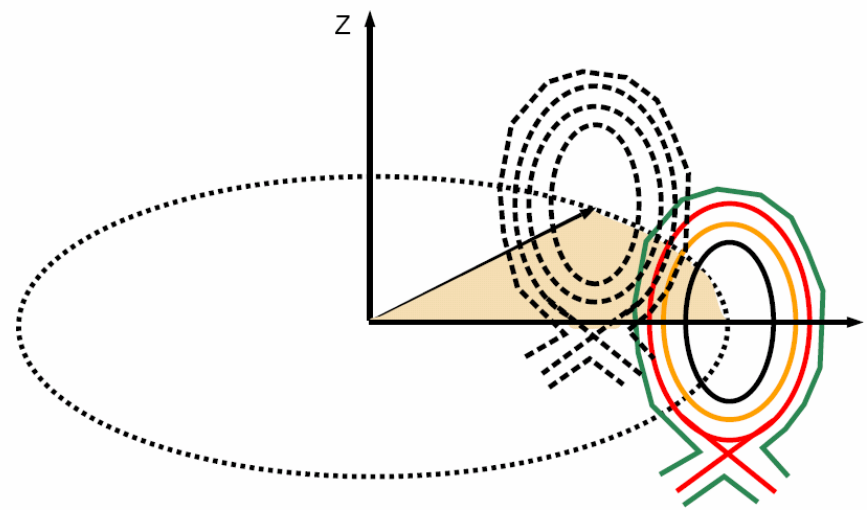
$$\begin{aligned} N_\alpha(\mathbf{x},t) &\equiv \frac{2\pi}{M_\alpha}\int B_\parallel^* d\bar{v}_\parallel d\bar{\mu} F_\alpha, \\ n_e(\mathbf{x},t) &\equiv \frac{2\pi}{m_e}\int B_\parallel^* dv_\parallel d\mu f_e, \\ p_{\perp\alpha} &= \pi B \int dv_\parallel d\bar{\mu}(v_\perp^2 F_\alpha), \\ T_{\perp\alpha} &= \frac{p_{\perp\alpha}}{N_\alpha(\mathbf{x},t)}. \end{aligned}$$

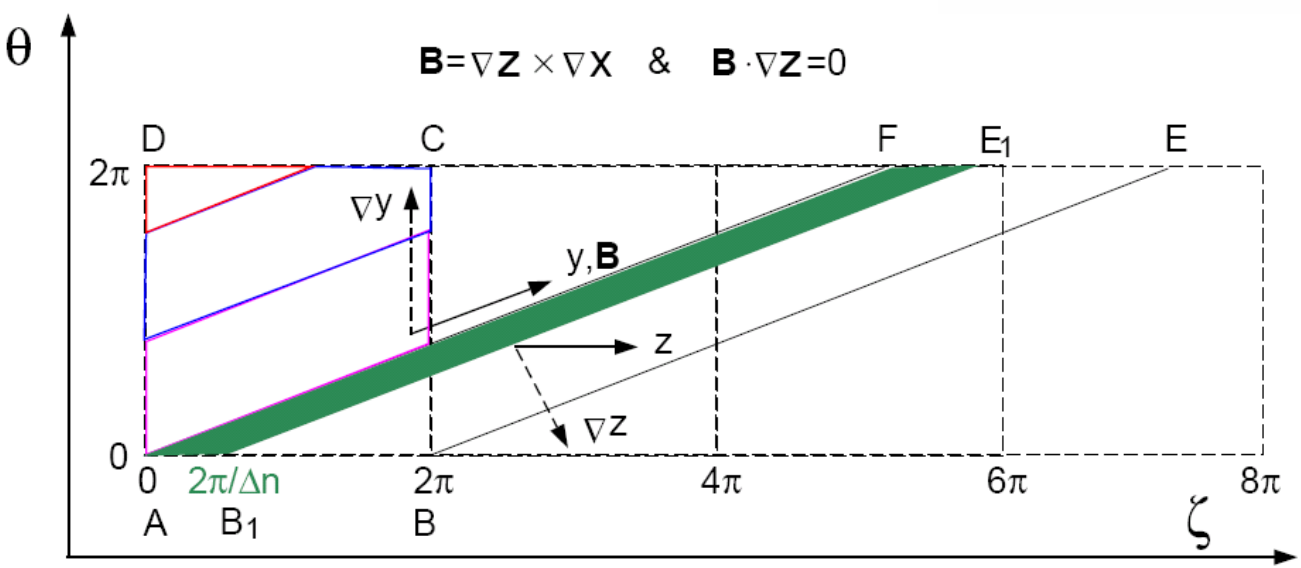
The n_α and $T_{\perp\alpha}$ are the normalization density and temperature. The ion gyroradius is $\rho_\alpha = \sqrt{2T_{\perp\alpha}/M_\alpha}/\Omega_\alpha$, the ion gyrofrequency is $\Omega_\alpha = Z_\alpha eB/M_\alpha c$, and the ion Debye length is $\lambda_{D\alpha}^2 = T_{\perp\alpha}/4\pi n_\alpha Z_\alpha^2 e^2$.

- ✓**Diamagnetic density is included;**
- ✓**It is fully nonlinear since the N_α and P_α are calculated from F_α;**
- ✓**If the first-order Pade approximation to $\Gamma_0 = 1/(1 + b)$ for the modified Bessel function is used, then the same field solve will be used in the arbitrary wavelength regime.**

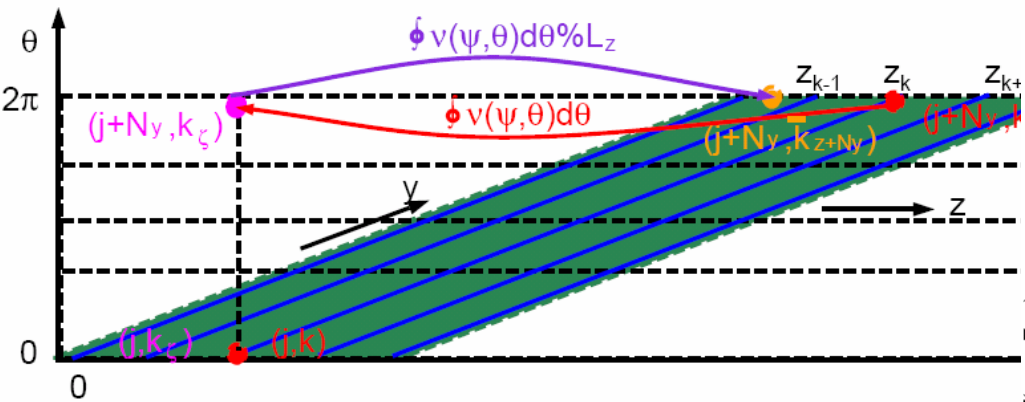
Field-aligned coordinates for high-n turbulence simulations

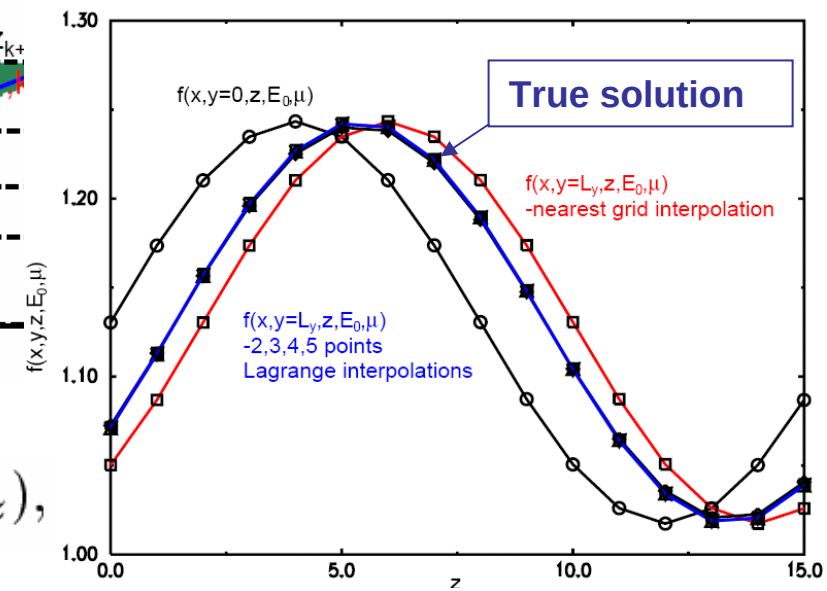
$$
\begin{aligned}
x &= \psi - \psi_s, \\
y &= \theta, \\
z &= \zeta - \int_{y_0}^{y} \nu(x,y)\,dy.
\end{aligned}
$$





Twisted parallel boundary condition using 5 points Lagrange interpolation in z





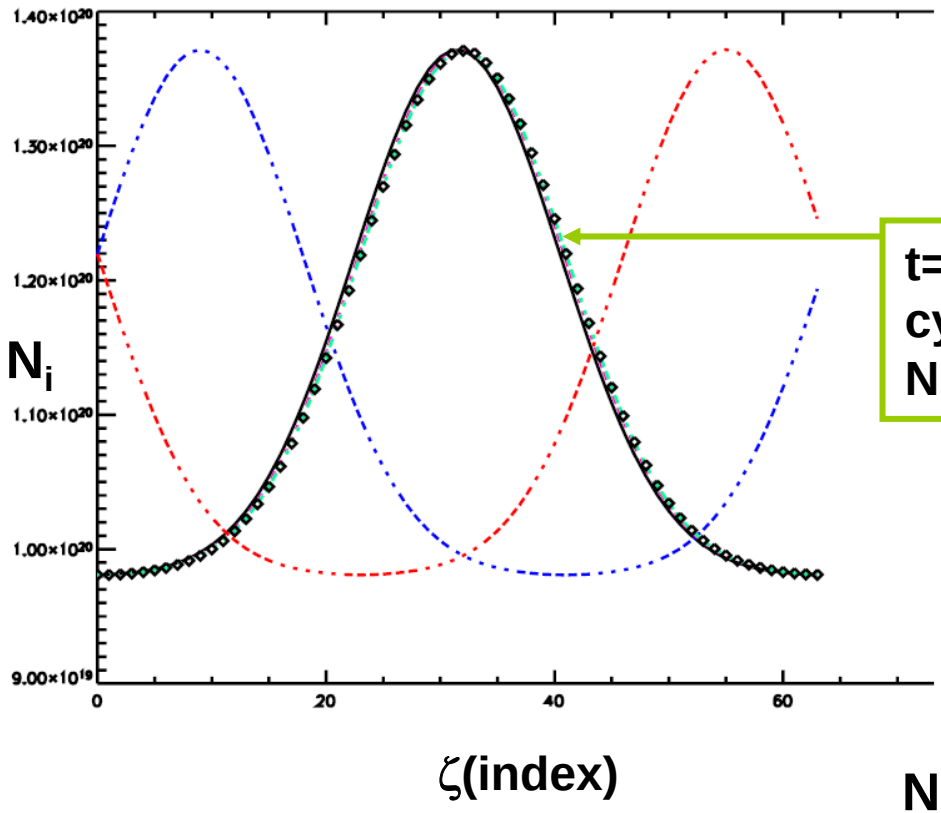
$$F(i_x, j_y + N_y, \bar{k}_{z+N_y}) \quad = \quad F(i_x, j_y, k_z),$$

$$\bar{k}_{z+N_y} \quad = \quad \begin{cases} k_z - \dfrac{[\oint \nu(x,y)dy]\%L_z}{dz}, & k_z > \dfrac{[\oint \nu(x,y)dy]\%L_z}{dz}, \\ z_k - \dfrac{[\oint \nu(x,y)dy]\%L_z}{dz} + N_z, & k_z < \dfrac{[\oint \nu(x,y)dy]\%L_z}{dz} \end{cases}$$

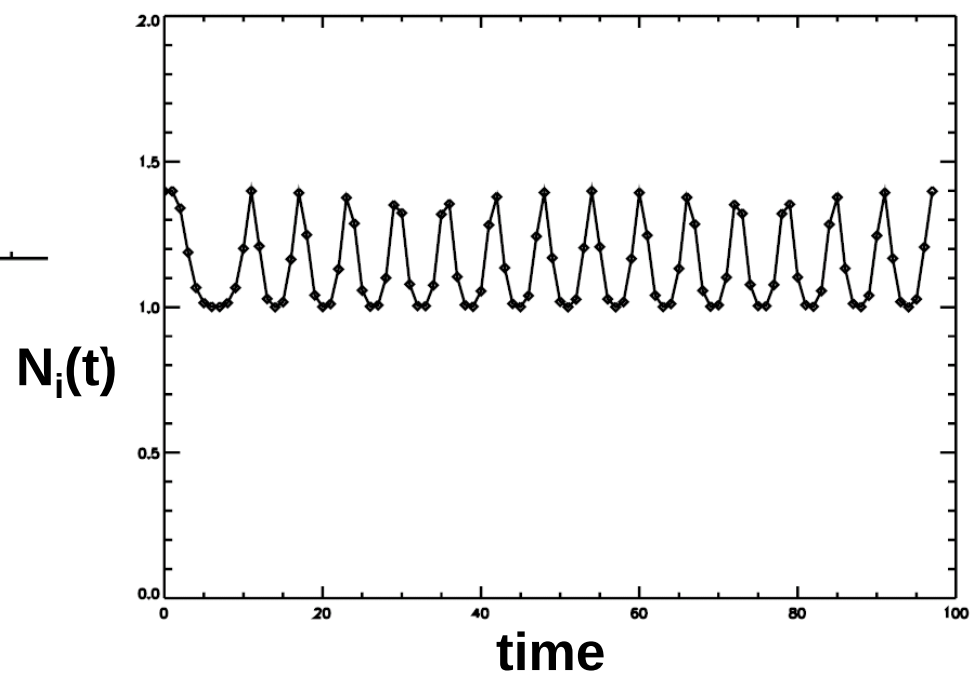
✓**The same procedure is used for the radial difference**

Toroidal convection using 5th order Weno scheme

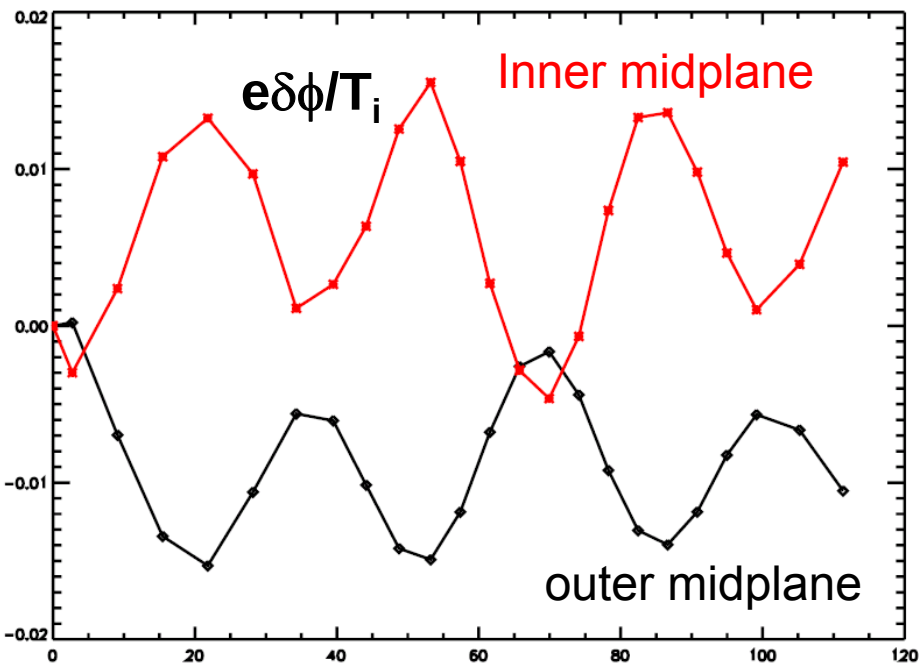
---an accurate convection algorithm



t=0, t=1st cycle and t=14th cycles are overlapped, Nz=64



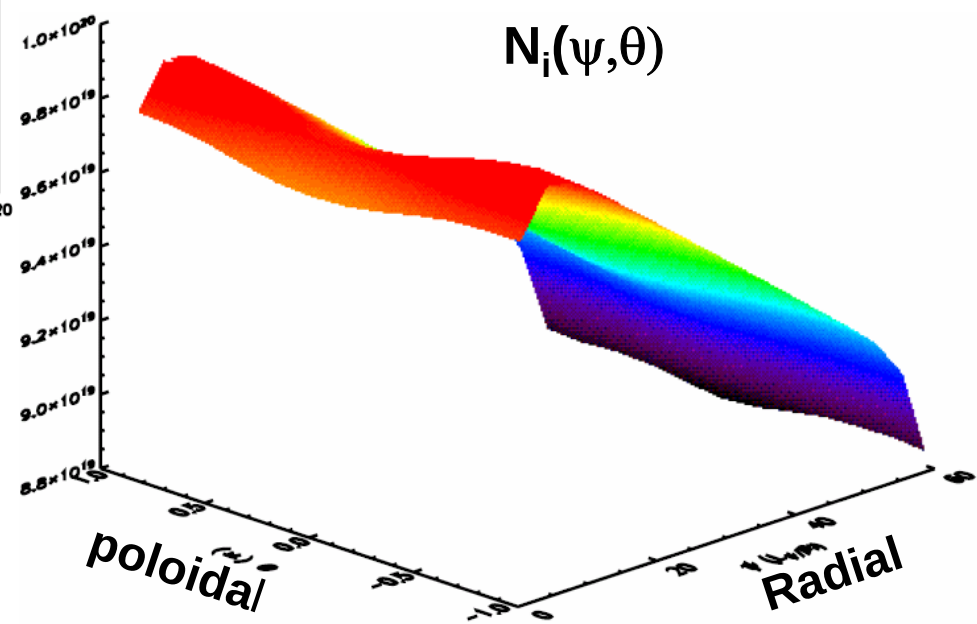
First 5D tempest results
Drift wave frequency agrees with theory



Time($t_n=V_{ti}/R$)

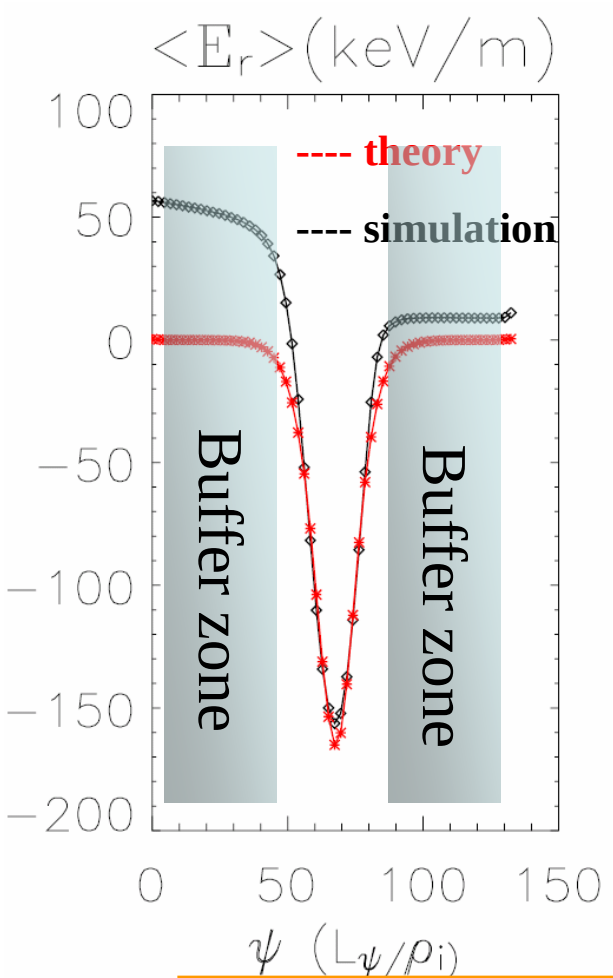
- ✓ $\omega_{*n}t_n$~0.23 → ω^{sim}/ω_{*n} ~1.11
- ✓ **16x12x16x25x50**
- ✓ $k_\theta\rho_i$~0.0036

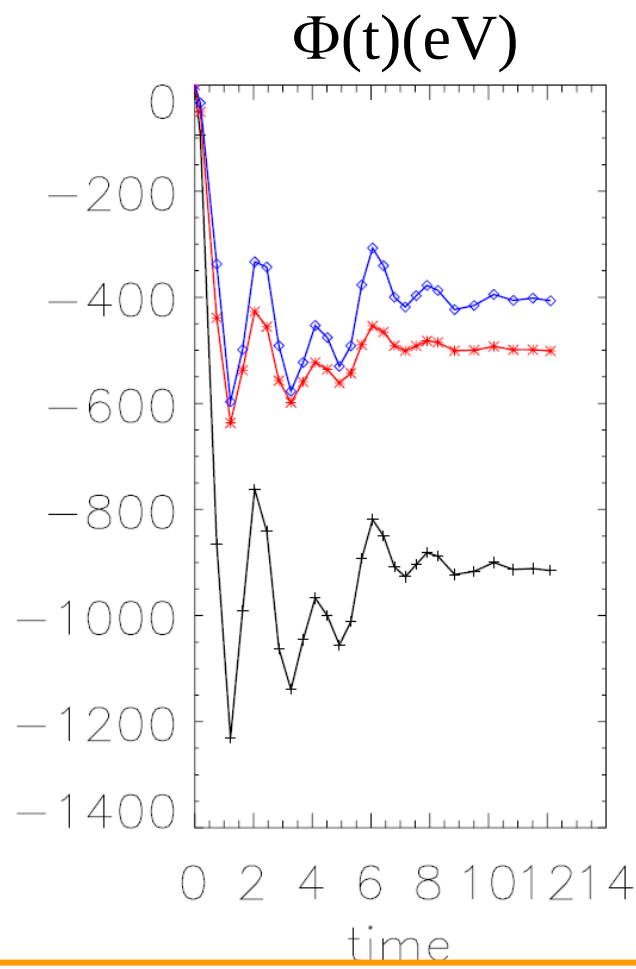
- ✓ **Boltzmann e^- model**
- ✓ **Flat T_i profile**
- ✓ **10% density variation**
- ✓ **Circular geometry,**
- ✓ **q=3, ε=0.02**



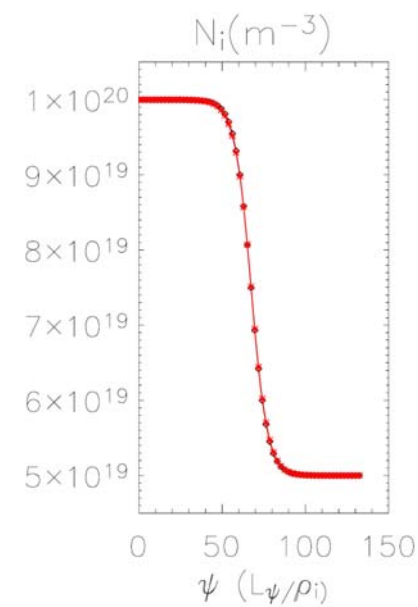
Neoclassical Er

TEMPEST yields a self-consistent neoclassical E_r, which agrees with theory





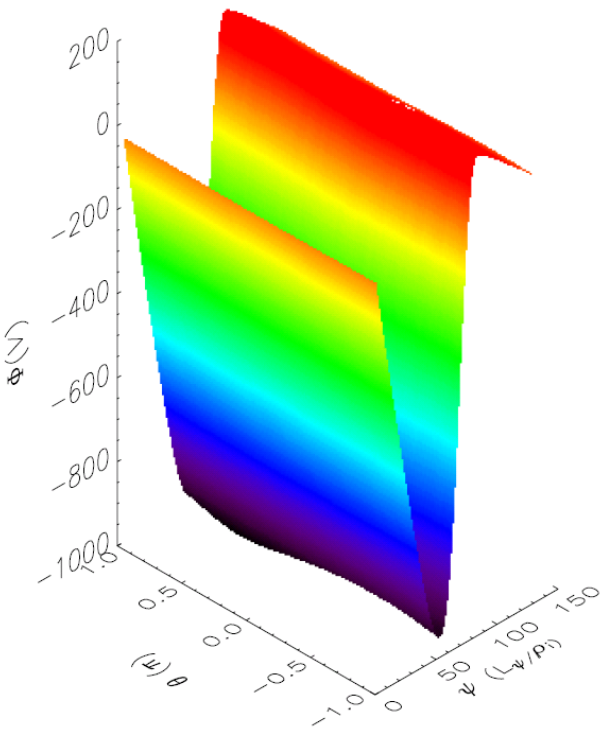
- ✓In Circular geometry
- ✓Large aspect ratio
- ✓Boltzmann electrons
- ✓Small collision
- ✓Radial boundary buffer zones used

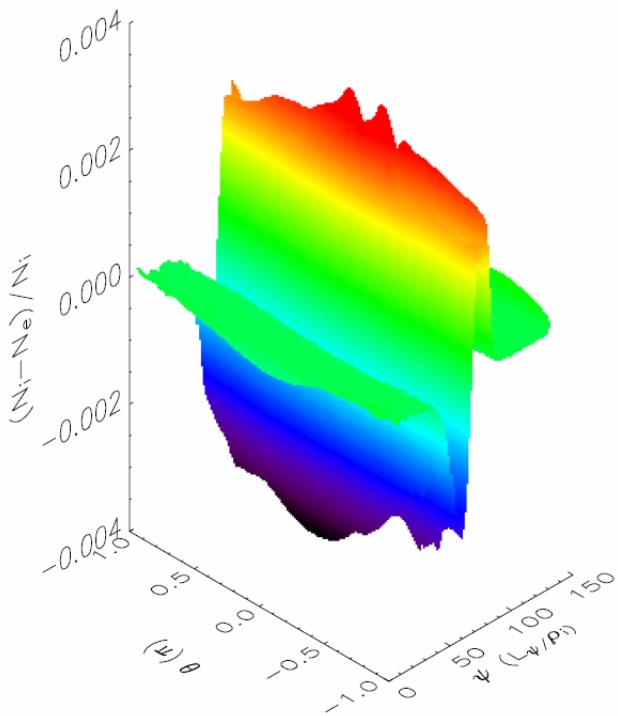


$$< U_{||} >= \frac{IT_i}{\Omega\psi' m_i}[(k-1)\frac{\partial \ln T_i}{\partial r} - \frac{\partial \ln n_i}{\partial r} + \frac{e}{T}E_r]$$

TEMPEST shows E_r generation by neoclassical polarization in a steep gradient region

- ✓**Relative maximum charge separation is only 0.4%**
- ✓**A plausible mechanism for pedestal E_r: neoclassical drive, $v_d \cdot \nabla F$**
- ✓**$L_\theta >> L_r$**





Summary

- Completed 5D in Ring geometry
- 95% completed 5D in divertor geometry
  - Extended field solve to 3D
    - To be completed for geometric coefficients in Poisson equation
- Gyroaverage
  - Needed to extend to 5D
- 3D domain decomposition
  - To be completed for parallelization in binormal direction.
- 5D results in Ring geometry
  - Drift wave frequency matches theory as expected.
  - Test runs for ITG turbulence are underway
- TEMPEST yields a self-consistent neoclassical E_r, which agrees with theory